Applied Soft Computing 47 (2016) 438–448

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

Relations between complex vague soft sets

Ganeshsree Selvachandran [a,*], P.K. Maji [b], Israa E. Abed [c], Abdul Razak Salleh [c]

[a] Department of Actuarial Science and Applied Statistics, Faculty of Business & Information Science, UCSI University, Jalan Menara Gading, 56000 Cheras, Kuala Lumpur, Malaysia
[b] Department of Mathematics, B.C. College, Asansol, West Bengal, India
[c] School of Mathematical Sciences, Faculty of Science and Technology, Universiti Kebangsaan Malaysia, 43600 UKM Bangi, Selangor DE, Malaysia

ARTICLE INFO

Article history:
Received 11 April 2016
Received in revised form 27 May 2016
Accepted 31 May 2016
Available online 21 June 2016

MSC:
08A99
08A72

Keywords:
Complex vague soft set
Relations between complex vague soft sets
Complex fuzzy set
Vague soft set
Fuzzy set

ABSTRACT

Complex vague soft sets are essentially vague soft sets characterized by an additional parameter called the phase term which is defined over the set of complex numbers. In this study, we introduce and discuss the relations between complex vague soft sets. We present the definitions of the Cartesian product of complex vague soft sets and subsequently that of complex vague soft relations. The definition of the composition of complex vague soft sets is also provided. The notions of symmetric, transitive, reflexive and equivalence complex vague soft relations are then proposed and the algebraic properties of these concepts are verified. The relation between complex vague soft sets is then discussed in the context of a real-life problem: the relation between the financial indicators of the Chinese economy which are characterized by their degrees of influence on the financial indicators of the Malaysian economy, and the time required for the former to affect the latter. Interpretations of the results obtained from this example are then proposed by relating them to recent significant real-life events in the Chinese and Malaysian economies.

1. Introduction

The complex number set allows us to solve many problems that traditionally cannot be solved by using the real number set, such as improper integrals that represent electrical resistance in the field of engineering. Thus applying soft, fuzzy and hybrid sets to complex numbers is an essential step to incorporate the advantages of complex numbers into the notion of soft sets, fuzzy sets, and their generalizations. This work initiated by Ramot et al. [1], who introduced the concept of complex fuzzy sets which extend the notion of fuzzy sets, are made possible by adding a phase term that describes the periodicity of the elements with respect to time. The range of Ramot's complex fuzzy sets is not limited to [0,1] but rather extends to the unit circle in the complex plane. The theory behind the development of complex fuzzy sets says that in many instances a second dimension must be added to the expression of the membership value of an element or object which is particularly important in situations wherein the elements of the set vary with time. Examples include meteorological time series such as sunspot cycles which describe the number of sunspots that appear on the surface of the sun as a function of time and economic time series such as the fluctuation of stock prices on a daily or hourly basis or the effects of certain financial factors on the economy of a country or a region. Among other notable research in this relatively unexplored area are the application of complex fuzzy sets to traditional fuzzy logic via the introduction of complex fuzzy logic by Ramot et al. [2] and Dick [3] who further expanded the theory of complex fuzzy logic by introducing several new operators pertaining to this theory.

The study of uncertainties and nonlinear problems and by extension, the modelling of uncertainties and nonlinear problems has always been a major area in the study of mathematics. Over the years, many techniques and methods have been proposed as tools to be used to find the solutions of problems that are nonlinear or vague in nature, with every method introduced superior to its predecessors. The study of nonlinear problems is of particular interest to engineers, physicists, mathematicians and other scientists as most systems in the real world are inherently nonlinear in nature and often appear as chaotic, unpredictable and sometimes even counterintuitive. Applied mathematical techniques and artificial intelligence are the most commonly used methodologies used to handle perturbation and chaotic

* Corresponding author.
E-mail addresses: ganeshsree86@yahoo.com (G. Selvachandran), pabitra_maji@yahoo.com (P.K. Maji).

http://dx.doi.org/10.1016/j.asoc.2016.05.055
1568-4946/

behavior of nonlinear problems and find accurate solutions to these problems. Among the notable research done in this area are due to Argyros & George [4] and Argyros & Gonzalez [5]. In Ref. [4], the authors presented a local convergence analysis for a family of Steffenson-type fourth order methods in order to obtain approximate solutions for nonlinear equations and proved that the method proposed here has the ability to be extended under weaker hypotheses. A local convergence analysis was also presented in Ref. [5], but for an improved Jarrett-type method of at least five to obtain approximate solutions for nonlinear equations in a Banach space setting. It was proven that the convergence ball and error estimates introduced here which was derived using the hypotheses up to the first Frechet derivative is superior and more widely applicable in finding approximate solutions to nonlinear problems as opposed to methods introduced in previous studies, many of which uses hypotheses up to the third Frechet derivative. Besides that, one of the most recent research done in the study of complex dynamics pertaining to nonlinear problems is due to Amat et al. [6] who utilized the dynamics of a two-step Newton-type method for solving nonlinear equations and systems and went on to prove that there exists significant characteristics of attraction, chaos and perturbation for some choices of the damping factor.

Another commonly used method in handling uncertainties and representing incomplete and unreliable data is soft computing, which was born as a direct result of the establishment of soft set theory by Molodtsov [7]. Soft computing is in fact, a collection of methodologies and techniques that aim to push the limits of tolerance for imprecision, uncertainty and partial truths, with the objective of obtaining robust and accurate solutions with a reasonable amount of computation. These soft computing techniques are widely used in the modelling of imprecision, vagueness and subjective data, which is another major area of study in both pure and applied mathematics. Since its inception, soft computing methods have been combined with many of the classical mathematical theories that are used to handle uncertainties with the most prominent tools being fuzzy sets, soft sets and their generalizations.

In recent years, there has been a significant increase in the application of artificial intelligence or applied mathematics techniques which are combined with mathematical models constructed using fuzzy sets, soft sets and/or hybrid models of these two sets. It is also worth noting that there have been an exponential increase in the utilization of applied mathematical techniques to aid with and improve the efficiency of computation in several prominent areas in the study of fuzzy sets and systems. This is particularly true in the study of fuzzy differential equations and fuzzy partial differential equations, where the recent trend in this area include among others, using fuzzy differential transform methods and Laplace-Fourier transforms in finding the solutions to fuzzy Volterra integro differential equations (see Refs. [8–10]). In the same vein, in this paper, we apply soft computing techniques to develop a better model to evaluate the degree of interaction between two hybrid models which are used to represent two-dimensional data that is subjective, humanistic, vague and subject to perceptional differences.

Selvachandran et al. [11] introduced the concept of complex vague soft sets (CVSSs) which combine the key features of soft and complex fuzzy sets. The CVSS model is an improved hybrid model of complex fuzzy sets that can (1) represent two-dimensional information i.e. information about a parameter and its periodicity by virtue of a phase term; (2) be more expressive in capturing the vagueness and unreliability of data by virtue of the vague soft set used in this model, which allows interval-based membership values to overcome the problem of assigning suitable membership values to the elements in a set compared with fuzzy or intuitionistic fuzzy sets and better corresponds to the intuition of representing vague data and (3) provide a more adequate parametrization tool that can represent the parameters of a problem in a more comprehensive manner. The CVSS model possesses all of these abilities because it embodies all of the features of complex fuzzy sets with the added advantage of vague sets and the adequate parameterization characteristic of soft sets. We refer the readers to Ref. [11] for further information on CVSSs and their properties.

In the present work, we introduce and study the various types of relations between CVSSs. The relations between fuzzy sets, soft sets and their many generalizations are a topic that has been extensively studied. However, the most significant shortcoming of these relations is their inability to capture information pertaining to the time frame of the intersection between the parameters, which is a very important component of most real-life situations. Improving this deficiency in our understanding of the relations between various hybrids of fuzzy and soft sets formed the motivation for this work. As such, we herein introduce the complex vague soft relation between CVSSs, which represents the degree of presence or absence of interaction and the phase of the interaction between the elements of the CVSSs. We then provide verifications of some fundamental properties of this concept, present the application of this concept to an economics example and support it with real-life events in the Malaysian and Chinese economies.

2. Preliminaries

In this section, we recapitulate some of the important concepts pertaining to soft sets and the hybrid structures of soft sets such as vague soft sets and complex fuzzy sets that are relevant to this paper.

Definition 2.1 *([12])*. Let X be a space of points (objects) with elements of denoted by x. A *vague set* V in X is characterized by a truth-membership function $t_V : X \to [0, 1]$ and a false-membership function $f_V : X \to [0, 1]$. The function $t_V(x)$ gives the lower bound of the grade of membership of x derived from the evidence for x and $f_V(x)$ gives the lower bound of the negation of x derived from the evidence against x. The functions $t_V(x)$ and $f_V(x)$ each associate a real number in the interval [0, 1] with each point in X, where $0 \le t_V(x) + f_V(x) \le 1$. This approach bounds the grade of membership of x to a closed subinterval $[t_V(x), 1 - f_V(x)]$ of [0, 1].

When X is continuous, the vague set V can be written as:

$$V = \frac{\int [t_V(x), 1 - f_V(x)]}{x}, \quad \text{where} x \in X.$$

When X is discrete, V can be written as:

$$V = \sum_{i=1}^{n} \frac{[t_V(x_i), 1 - f_V(x_i)]}{x_i}, \quad \text{where} x_i \in X.$$

By using vague sets, the accuracy of our knowledge of x is immediately clear through the uncertainty, which is characterized by the difference between $1 - f_V(x)$ and $t_V(x)$. If this difference is small, then our knowledge of x is accurate. Conversely, if this difference is large,