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Uncertain multi-attributes decision making method based on interval number with probability distribution weighted operators and stochastic dominance degree

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ABSTRACT

In real world decision making problems, real numbers, random numbers, and interval numbers are often used simultaneously to express the attribute values of alternatives. To solve these uncertain multi-attribute decision making problems, we propose a definition of interval number with probability distribution (INPD). This definition gives a uniform form for real numbers, interval numbers, and random numbers. Under certain conditions, an INPD can degrade to one of the three number forms. We then propose three weighted operators that aggregate opinions expressed by INPD. Furthermore, we propose a new stochastic dominance degree (SDD) definition based on the idea of almost stochastic dominance to rank two INPD. The new definition overcomes defects in traditional stochastic dominance methods. It takes all stakeholders' preferences into account and can measure both standard and almost SDDs. For real numbers and interval numbers, results derived from SDD are consistent with traditional methods. On this basis, a method using INPD weighted operators and SDD is proposed to solve uncertain multi-attribute decision making problems. Finally, three numerical examples are given to illustrate the applicability and effectiveness of the proposed method.

1. Introduction

Uncertain multi-attributes decision making (MADM) aims to find the most desirable solution from a set of feasible alternatives, where the information provided by decision makers (DMs) is usually uncertain due to the increasing complexity of the socio-economic environment, inherent restrictions of measuring technology, and the vagueness of human thinking. There are many uncertain MADM problems in real world situations. In these decision making processes, DMs cannot estimate attribute values with exact numerical values, but usually use some uncertain methodology [28]. Interval numbers and random numbers are usually used for measuring uncertainty. Interval numbers are suitable to measure the subjective uncertainty derived from the vague and subjective nature of human thinking [29]. While random numbers suit for the objective one derived from the complexity of object and the restriction of measuring technology [23]. Many methods have been developed to handle MADM problems using interval numbers or random numbers ([2,3,7,10,24,26,30]; [32]). However, in some evaluation problems, real numbers, interval numbers, and random numbers may be used simultaneously to express values of different attributes. Some evaluation data may have been collected from measuring instruments or a historical database and expressed using real and/or random numbers. In addition, experts may also be invited to evaluate some subjective attributes. Due to the vagueness of human thinking, these attribute values are generally expressed by an interval number. In such cases, methods using a single number form may be not applicable.

One possible way to solve this problem is to translate the three forms of numbers into a uniform format. Real numbers can be considered as a special form of an interval or random number. The key to problem solving is to find the relation between interval numbers and random numbers. The possibility degree method is a simple and effective method used to rank interval numbers ([6]; [11,25]). The method assumes that an interval number is a uniform distribution variable with closed interval support. In fact, most existing research regarding interval numbers is based on this assumption [20]. When additional information about the nature of the uncertainty is obtained, other distributions, such as normal distribution, can be used to describe the probability on the interval [1,18]. In practice, interval numbers are usually used to express a DM's uncertain opinion and are considered as continuous sets [19]. However, random numbers are generally considered an extension of real numbers and numerical values. Our

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new uniform format must contain characteristics of both types of numbers.

In some situations, such as in group decisions, DMs are simply agents of all the stakeholders. Decisions should be made based on the preferences of all stakeholders, not those of the agents. In such cases, the elicitation of a unique probability or utility function may be difficult and the validity of its use is questionable [27]. One well-regarded method for comparing two alternatives with uncertain utility information is through stochastic dominance (SD). The SD method for comparing two alternatives with uncertain information has many advantages. It takes the difference of the stakeholders' utility function into account and compares the expected utility of alternatives in a pairwise manner, only making minimal assumptions regarding the utility function, and makes no assumption at all with respect to the particular probability distributions of returns [15]. However, there are certain obvious disadvantages of the method. First, SD rules are strong conditions. Sometimes, an SD relation between two alternatives does not exist [22]. Second, SD relations are qualitative rather than quantitative. The verification of SD relations is not sufficient to accept strict preferences if the alternatives differ insignificantly [17].

There have been many attempts by scholars to solve these problems. One potentially feasible solution is the relaxation of SD rules. Leshno and Levy [8] defined the concept of almost stochastic dominance (ASD). It is a form of SD which holds for most, but not all, of the utility functions in a given class. Some utility functions are deemed 'extreme' and it is assumed that they do not represent the preferences of any real world stakeholder. Leshno and Levy [8] suggested that we should rule out such utility functions. On the other hand, some scholars considered the issues regarding the quantification of SD rules. Some alternate SDD definitions have been proposed ([21,33]). However, most definitions have no economic meaning; they are simply ratios of areas enclosed by cumulative distribution function curves. The results obtained by these methods may be dubious. Besides, methods that can measure both SD and ASD degrees are still scarce.

In this paper, we first propose a definition and operators for interval numbers with probability distribution (INPD) (Section 2). This is a uniform form definition for real numbers, interval numbers, and random numbers. Under certain conditions, INPD can degrade to the three different number forms. Three INPD weighted operators are also proposed based on various aggregation ideas for different decision circumstances. Then, the idea of SD is introduced to rank two INPDs (Section 3). To overcome the defects of traditional SD rules, we propose a new SDD definition based on the idea of ASD. This definition can be used to measure degrees of SD and ASD and has a clear economic meaning. For an alternative, its SDD is determined by its own performance and support from stakeholders. We then propose a method based on INPD weighted operators and SDD to solve uncertain MADM problems (Section 4). Furthermore, numerical examples and comparative analysis are shown in Section 5. Finally, the paper is concluded in Section 6.

2. INPD basic concepts and aggregation operations

Definition 1. Let $\bar{a} = [a^l, a^u] = \{x | a^l \le x \le a^u; a^l, a^u \in R\}$, and $f(x)$ be probability density functions on the range $[a^l, a^u]$, where $\int_{a^l}^{a^u} f(x)dx = 1$, and $\forall x \in [a^l, a^u]$, $f(x) \ne 0$; then, $(\bar{a}, f(x))$ or $([a^l, a^u], f(x))$ is an interval number with probability distribution (INPD). Hereinafter, $\tilde{a}$ is used to denote $(\bar{a}, f(x))$ for brevity.

In particular, when the closed interval $[a^l, a^u]$ is extended to $(-\infty, +\infty)$, its INPD degrades to a random number. When the distribution function $f(x)$ has a uniform distribution, the INPD degrades to an interval number. When $a^l = a^u$, the INPD degrades to a real number. The basic INPD operations are proposed as follow.

To simplify the calculations, throughout this paper, let $\Omega = \{\tilde{x} | \tilde{x} = ([x^l, x^u], f(x)), x^l \le x \le x^u, x^l, x^u \in R\}$ be the set of all INPD.

Definition 2. Let $\tilde{a}, \tilde{b} \in \Omega$, $\tilde{a} = ([a^l, a^u], f_1(x_1))$, $\tilde{b} = ([b^l, b^u], f_2(x_2))$, $\lambda \in R$ and $\lambda \ne 0$, then:

$$\tilde{a} \oplus \tilde{b} = ([a^l + b^l, a^u + b^u], f_\oplus(x)) \tag{1}$$

$$\tilde{a} \otimes \tilde{b} = ([\min(a^l b^l, a^l b^u, a^u b^l, a^u b^u), \max(a^l b^l, a^l b^u, a^u b^l, a^u b^u)], f_\otimes(x)) \tag{2}$$

$$\lambda \cdot \tilde{a} = \left([\lambda a^l, \lambda a^u], f\left(\frac{x}{\lambda}\right)\right) \tag{3}$$

where $f_\otimes(x) = \iint_{x_1 \cdot x_2 = x} f_{12}(x_1, x_2)dx_1dx_2$, $f_\oplus(x) = \iint_{x_1 + x_2 = x} f_{12}(x_1, x_2)dx_1dx_2$; $f_{12}(x_1, x_2)$ is the joint distribution of $f_1(x_1)$ and $f_2(x_2)$. When $\tilde{a}$ and $\tilde{b}$ are independent, $f_\otimes(x) = \iint_{x_1 \cdot x_2 = x} f_1(x_1) \cdot f_2(x_2)dx_1dx_2$, $f_\oplus(x) = \iint_{x_1 + x_2 = x} f_1(x_1) \cdot f_2(x_2)dx_1dx_2$. In decisions, INPDs are used to express attribute values of alternatives given by different DMs. In such cases, the INPD are considered to be independent. In the rest of this paper, we assume INPDs are independent.

In the above operations, an INPD is deemed to be an extension of a real number or a numerical value. However, in practice, INPDs can also be thought of as uncertain opinions given by DMs. In such cases, for an INPD $\tilde{x} = ([x^l, x^u], f(x))$, the interval $[x^l, x^u]$ reflects the value range of x. The function $f(x)$ describes the likelihood decided by the DMs for x to take on a given value. For a given value x_0, different DMs may have different opinions. The common opinion should depend on their weights and their personal opinions. The other two operators are given as follows based on the weighted sum model:

Definition 3. Let $\tilde{a}, \tilde{b} \in \Omega$, $\tilde{a} = ([a^l, a^u], f_1(x_1))$, $\tilde{b} = ([b^l, b^u], f_2(x_2))$, $\alpha, \beta \in [0, 1]$ are the weights of $\tilde{a}$ and $\tilde{b}$,with $\alpha + \beta = 1$, then:

$$\tilde{a} \bigcup \tilde{b} = \left([a^l, a^u] \bigcup [b^l, b^u], f_\cup(x)\right) \tag{4}$$

$$\tilde{a} \bigcap \tilde{b} = \left([a^l, a^u] \bigcap [b^l, b^u], f_\cap(x)\right) \tag{5}$$

where $f_\cup(x) = \alpha \cdot f_1(x) + \beta \cdot f_2(x)$,

$$f_\cap(x) = \begin{cases} \alpha \cdot \varepsilon_1 \cdot f_1(x_1) + \beta \cdot \varepsilon_2 \cdot f_2(x_2) & [a^l, a^u] \bigcap [b^l, b^u] \ne \emptyset \\ 0 & [a^l, a^u] \bigcap [b^l, b^u] = \emptyset \end{cases}$$

$$\varepsilon_1 = \frac{1}{\int_{[a^l, a^u] \bigcap [b^l, b^u]} f_1(x_1)dx_1}, \quad \varepsilon_2 = \frac{1}{\int_{[a^l, a^u] \bigcap [b^l, b^u]} f_2(x_2)dx_2}.$$

ε_1, ε_2 are the conversion coefficients used to gather the DMs' personal opinion to the new intersection.

From Definition 3, it is easy to obtain the following properties:

(1) $\tilde{a} \bigcup \tilde{a} = \tilde{a}$, $\tilde{a} \bigcap \tilde{a} = \tilde{a}$.
(2) $\forall x_0 \in [a^l, a^u] \cup [b^l, b^u]$, $\min(f_1(x_0), f_2(x_0)) \le f_\cup(x_0) \le \max(f_1(x_0), f_2(x_0))$.
(3) $\forall x_0 \in [a^l, a^u] \cap [b^l, b^u]$, $\min(\varepsilon_1 f_1(x_0), \varepsilon_2 f_2(x_0)) \le f_\cap(x_0) \le \max(\varepsilon_1 f_1(x_0), \varepsilon_2 f_2(x_0))$.
(4) $\tilde{a} \bigcup \tilde{b}, \tilde{a} \bigcap \tilde{b} \in \Omega$.

Based on the above operators, this paper proposes three INPD weighted operators, which represent three different aggregation ideas respectively, compromise, consensus, and acceptance.