Knowledge-Based Systems 000 (2016) 1–8

Contents lists available at ScienceDirect

Knowledge-Based Systems

journal homepage: www.elsevier.com/locate/knosys

Comparisons of probabilistic linguistic term sets for multi-criteria decision making

Chengzu Bai [a,b], Ren Zhang [a,b,*], Longxia Qian [a,b], Yaning Wu [c]

[a] Research Center of Ocean Environment Numerical Simulation, Institute of Meteorology & Oceanography, PLA University of Science and Technology, Nanjing 211101, China
[b] Collaborative Innovation Center on Forecast Meteorological Disaster Warning and Assessment, Nanjing University of Information Science & Technology, Nanjing 210044, China
[c] Research Center of Software Engineering, Institute of Information System, PLA University of Science and Technology, Nanjing 211101, China

ARTICLE INFO

Article history:
Received 29 September 2016
Revised 30 November 2016
Accepted 19 December 2016
Available online xxx

Keywords:
Probabilistic linguistic term sets
Multi-criteria decision making
Possibility degree formula
Fuzzy ranking

ABSTRACT

The theory of probabilistic linguistic term sets (PLTSs) is very useful in dealing with the multi-criteria decision making (MCDM) problems in which there is hesitancy in providing linguistic assessments; and PLTSs allow experts to express their preferences on one linguistic term over another. The existing approaches associated with PLTSs are limited or highly complex in real applications. Hence, the main purpose of this paper is to establish more appropriate comparison method and develop a more efficient way to handle with MCDM problems. We first put forward a diagram method to analyze the structures of PLTSs and develop the visualized way for readers to comprehend. Then a possibility degree formula is given for ranking PLTSs. Based on the new comparison method and the theory of the fuzzy preference relation, an efficient decision-making framework is proposed to solve real-life problems under linguistic environment. Conventional TOPSIS methods combined with PLTSs are also included for comparison. All results demonstrate the practicality of the new framework. Finally, we also seek out relationship between PLTSs and hesitant fuzzy linguistic term sets (HFLTSs), and compare the new formula with the similar approaches to HFLTSs' rating.

1. Introduction

In practical multi-criteria decision making (MCDM) problems, experts commonly prefer to express their preferences by linguistic terms, such as "perfect", "high" or "poor", instead of quantitative evaluations due to the qualitative nature of many criteria or the dilemma of selecting appropriate numerical values. Therefore, many linguistic approaches have been developed and extended to model qualitative information or improve its computation. To facilitate readers' understanding of the existing main linguistic models, we first briefly summarize their own characteristics in Table 1, and then discuss about differences in detail as follows.

From Table 1, there are three well-known linguistic models, i.e. the semantic model [1], the symbolic model [2] and the linguistic 2-tuple model [3,4], which have been widely applied to different disciplines [5–9]. Later on, Herrera et al. [10,11] established a series of group decision making methods using linguistic term sets (LTSs). Xu [12] proposed a programming model for multi-attribute decision making under linguistic setting, and introduced the uncertain linguistic ordered weighted geometric (LOWG) and the induced uncertain LOWG operators [13] to deal with linguistic preference relations.

The aforementioned models are suitable for the situation which limits decision makers (DMs) to provide only single linguistic term as their assessments. In order to be easy for experts to express preferences on an alternative or a candidate, Rodriguez et al. [14] proposed a new concept of hesitant fuzzy linguistic term sets (HFLTSs) motivated by hesitant fuzzy sets [15], which allow DMs to use several possible LTSs to describe the priority over objectives. Then tremendous research has been developed to aggregate HELTSs information [16,17], compare HFLTSs [17], ease the computing process [18] and be combined with appropriate decision making models to solve more complex practical issues [19,20]. However, all possible linguistic evaluations provided by experts have equal weights or importance in most of the current approaches about HFLTSs. Obviously, it may be not appropriate in real-life MCDM problems, since the DMs may prefer some LTSs to other ones so that the associated weights of these linguistic assessments should have different values. For example, if the form of weights are taken as probability distributions, then the evaluation sets

* Corresponding author.
E-mail address: zrpaper@163.com (R. Zhang).

http://dx.doi.org/10.1016/j.knosys.2016.12.020

Table 1
A summary on the linguistic models for decision making.

Different models	Number of linguistic terms	Probabilistic information
Semantic Model [1]	Single	N/A
Symbolic Model [2]	Single	N/A
Linguistic 2-tuple model [3–9]	Single	N/A
Conventional LTSs [10–13]	Single	N/A
HFLTSs [14–20]	Several	N/A
Proportional linguistic terms [21]	Two	A completely known proportion
Numerical Scale [22,23]	Several	A completely known probabilistic distribution
Possibility Distribution-Based HFLTSs [24]	Several	A completely known probabilistic distribution
Evidential reasoning [25,26]	Several	A completely known probabilistic distribution
PLTSs [27]	Several	A partially known probabilistic distribution

include not only several possible linguistic terms but also the probabilistic information, the ignorance of which may lead to erroneous results. Some approaches have been successfully proposed to consider distinguishing the preferences or weights of possible LTSs [21–26]. Pang et al. [27] summarized differences of such these methods and developed a new general concept, i.e. probabilistic linguistic term sets (PLTSs), to extend conventional LTSs. Compared with other representation approaches to modeling linguistic information, PLTSs not only allow DMs to provide several possible LTSs accompanied with probabilistic information over an alternative or object, but also can deal with the partially incomplete assessments.

As to the comparisons of PLTSs, Pang et al. [27] established a framework for ranking PLTSs. We note that Pang's comparison method is conducted by the score or the deviation degree of each PLTS according to the associated functions. The comparison results derived by this method generally order a sequence of PLTSs with absolute priorities, which seems not to meet common sense, because it may be unreasonable to say one PLTS is absolutely superior to the other if these two PLTSs have some common linguistic judgments. In addition, one of the weaknesses in conventional theories regarding PLTSs is that the computing process for MCDM problems is complex [27].

Aiming to overcome the deficiencies discussed above, our main purpose here is to develop a new suitable comparison method for PLTSs, and construct a more efficient framework to handle with complex MCDM problems. We implement a diagram method to analyze the structures of PLTSs and put forward a possibility degree formula for comparing PLTSs. The new comparison method, compared with the traditional one of Pang et al.'s [27], sufficiently consider the situation in which different PLTSs may have common linguistic terms. On the MCDM problems under linguistic environment, the new possibility degree formula is introduced to achieve the same acceptable performance with conventional methods but reducing the computational complexity.

The remainder of this paper is organized as follows. Section 2 reviews the background of the PLTSs and some basic operation laws. In Section 3, we propose the diagram method to describe the structures of PLTSs and the possibility degree formula is defined in detail. Then, Section 4 establishes the new framework to deal with MCDM problems based on the new proposed formula and the theory of the fuzzy preference relation. Section 5 presents a real-life case that illustrates the advantages of our methodology. Moreover, we provide a comparative analysis with the similar techniques for HFLTSs rating in Section 6. Finally, Section 7 closes this paper with some conclusions.

2. Preliminaries

In this section, we mainly review some basic concepts and operations of PLTSs. We first assume every LTS discussed in this paper is finite and ordered totally as proposed in [28,29].

Definition 2.1. Let a linguistic term set be $S=\{s_t|t=0,1,\ldots,g\}$ or $S=\{s_t|t=-\tau,\ldots,-1,0,1,\ldots,\tau\}$, with odd cardinality where the midterm represents the assessment of "approximately 0.5" or "indifference", and the rest linguistic labels are placed symmetrically around it.

2.1. The concept of PLTSs

Definition 2.2. [27]. Let $S=\{s_t|t=-\tau,\ldots,-1,0,1,\ldots,\tau\}$ be a LTS. Then a PLTS is defined as

$$L(p)=\left\{L^{(k)}(p^{(k)})|L^{(k)}\in S,\ r^{(k)}\in t,\ p^{(k)}\geq 0,\right.$$
$$\left. k=1,\ 2,\ldots,\ \#L(p),\ \sum_{k=1}^{\#L(p)}p^{(k)}\leq 1\right\}$$

where $L^{(k)}(p^{(k)})$ is the linguistic term $L^{(k)}$ associated with the probability $p^{(k)}$, $r^{(k)}$ is the subscript of $L^{(k)}$ and $\#L(p)$ the number of all linguistic terms in $L(p)$.

Definition 2.3. [30,31]. Let $S=\{s_t|t=-\tau,\ldots,-1,0,1,\ldots,\tau\}$ be a LTS, $L(p)$, $L_1(p)$ and $L_2(p)$ be three PLTSs, and λ be a positive real numbers. $\eta^{(k)}\in g(L)$, $\eta_1^{(i)}\in g(L_1)$, $\eta_2^{(j)}\in g(L_2)$ and $k=1,2,\ldots,\#L$, $i=1,2,\ldots,\#L_1$, $j=1,2,\ldots,\#L_2$, where g is the equivalent transformation function [31]:

$$g:[-\tau,\tau]\to[0,1],\ g(L(p))=\left\{\left[\frac{r^{(k)}}{2\tau}+\frac{1}{2}\right]\left(p^{(k)}\right)\right\}=L_\gamma(p),\ \gamma\in[0,1],$$
$$g^{-1}:[0,1]\to[-\tau,\tau],\ g^{-1}\left(L_\gamma(p)\right)=\left\{s_{(2\gamma-1)\tau}\left(p^{(\gamma)}\right)|\gamma\in[0,1]\right\}=L(p)$$

(1) $L_1(p)\oplus L_2(p)=g^{-1}(\bigcup_{\eta_1^{(i)}\in g(L_1),\eta_2^{(j)}\in g(L_2)}\{(\eta_1^{(i)}+\eta_2^{(j)}-\eta_1^{(i)}\eta_2^{(j)})(p_1^{(i)}p_2^{(j)})\})$;

(2) $L_1(p)\otimes L_2(p)=g^{-1}(\bigcup_{\eta_1^{(i)}\in g(L_1),\eta_2^{(j)}\in g(L_2)}\{(\eta_1^{(i)}\eta_2^{(j)})(p_1^{(i)}p_2^{(j)})\})$;

(3) $\lambda L(p)=g^{-1}(\bigcup_{\eta^{(k)}\in g(L)}\{(1-(1-\eta^{(k)})^{\lambda})(p^{(k)})\})$;

(4) $L^{\lambda}(p)=g^{-1}(\bigcup_{\eta^{(k)}\in g(L)}\{(\eta^{(k)})^{\lambda}(p^{(k)})\})$.

The following example is given to show the PLTS and its operational laws.

Example 2.1. Let $S=\{s_t|t=-3,\ldots,-1,0,1,\ldots,3\}$ be a LTS, $L_1(p)=\{s_{-1}(0.4),s_0(0.4),s_1(0.2)\}$ and $L_2(p)=\{s_1(0.6),s_2(0.4)\}$ be two PLTSs. And $\lambda=2$, then

(1)

$$L_1(p)\oplus L_2(p)=g^{-1}\left(\bigcup_{\eta_1^{(i)}\in g(L_1),\eta_2^{(j)}\in g(L_2)}\{(\eta_1^{(i)}+\eta_2^{(j)}-\eta_1^{(i)}\eta_2^{(j)})(p_1^{(i)}p_2^{(j)})\}\right),$$