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Granular reducts of formal fuzzy contexts

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ABSTRACT

Knowledge reduction is one of the key issues in knowledge discovery and data mining. During the construction of a concept lattice, it has been recognized that computational complexity is a major obstacle in deriving all the concept from a database. In order to improve the computational efficiency, it is necessary to preprocess the database and reduce its size as much as possible. Focusing on formal fuzzy contexts, we introduce in the paper the notions of granular consistent sets and granular reducts and propose granular reduct methods in the sense of reducing the attributes. With the proposed approaches, the attributes that are not essential to all the object concepts can be removed without loss of knowledge and, consequently, the computational complexity of constructing the concept lattice is reduced. Furthermore, the relationship between the granular reducts and the classification reducts in a formal fuzzy context is investigated.

1. Introduction

The theory of formal concept analysis (FCA), proposed by Wille [11,47], centers on the study of formal concepts and conceptual hierarchies. It has been employed to unravel, from relational information systems, hierarchical concepts organized as a lattice. Since its inception, FCA has been applied to many real-life problems including data mining, information retrieval, knowledge acquisition, software engineering, data base management systems and on other disciplines [8,10,18,22–25,28,33,45,57]. Over the years, FCA has been an important research area with appealing theoretical and practical issues.

FCA is formulated based on a formal context materialized as a set of objects, a set of attributes, and a binary relation usually taking the form of a binary table that relates the objects to the attributes with value 0 and 1. However, in many real-life problems, the binary relations are fuzzy rather than crisp. Thus, formal fuzzy contexts are more common than their crisp counterparts. For this reason, binary fuzzy relations are used to analyze Galois connections between objects and attributes. Burusco and Fuentes-Gonzáles [4] first examined FCA in a fuzzy setting, and they defined L-fuzzy concepts using implication operators. In recent years, many researchers have extended FCA theory by using the ideas from fuzzy logic reasoning or fuzzy set theory, and several generalizations of formal fuzzy concept have been made (please see [1,9,12,13,15,34,40,48]). On the other hand, Krajči [19] and Yahia et al. [53] independently proposed the "one-sided fuzzy concept", where each fuzzy concept (X, B) takes the form of "X is crisp and B is fuzzy", or "X is fuzzy and B is crisp". Zhang et al. [60] further constructed the "variable threshold concept lattices", i.e. crisp-fuzzy variable threshold concepts and fuzzy-crisp variable threshold concepts, in which the "one-sided fuzzy concept" becomes a particular case (threshold being equal to 1). One can refer [2] for a comprehensive survey and comparison of the existing approaches for fuzzy concept lattices.

Granular computing (GrC) is an approach for knowledge representation and data mining. A granule is a clump of objects (points) drawn together by some criteria. The main directions in the study of GrC are the construction of granules and computation with granules. The former deals with the formation, representation, and interpretation of granules, while the latter handles the utilization of granules in problem solving [38,51,54,55]. More recently, there has been an increasing interest in the study of GrC, and many methods and models have been proposed and studied [6,29,32,37,39,41,42,49,52,56,59].

It should be noted that a concept lattice is constructed by all the formal concepts combined with a hierarchical order of the concepts. At the bottom of a concept lattice structure are object concepts, and other concepts (contained in the concept lattice) can be represented as a join of some object concepts. Hence, the object concepts play an important role in the construction of concept

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lattice and can be viewed as a basic information granule in the concept lattice. Thus, concept lattice can be treated as a concrete model of GrC. Wu et al. [50] first examined the granular structure of concept lattices and applied it to knowledge reduction of formal contexts. Kang et al. [16,17] introduced GrC into FCA and ontology learning, and presented a unified model for concept-lattice building and rule extraction under fuzzy granularity and ontology model building, ontology merging and ontology connection at different levels of granulation.

Attribute reduction is an important issue in the discovery of knowledge in information systems. In terms of a formal context, attribute reduction searches for a minimal attribute subset that preserves the required properties. Interest in attribute reduction in FCA has rapidly increased in recent years [7,9,20,21,26,30,35,43,44,46,50,61]. Ganter and Wille [11] first introduced the notions of reducible attributes and reducible objects by reducing columns and rows in a formal context. Zhang and Wei [61] discussed attribute reduction in classical formal contexts, and formulated a reduction approach by using discernibility matrices and Boolean functions. Liu et al[31]. presented a reduction method for concept lattices based on rough set theory. In [46], Wang and Zhang proposed a reduction approach by keeping the meet-irreducible elements. Based on fuzzy K-means clustering, Kumar and Srinivas [20] put forward a method to reduce the size of a concept lattice by employing the corresponding object-attribute matrix. Shao et al. [44] formulated a knowledge-lossless approach to complexity reduction in formal decision contexts with which the complexity of concept lattice is substantially reduced. Li et al. [21] developed a rule-acquisition oriented framework of knowledge reduction for real decision formal contexts and formulated a reduction method by constructing a discernibility matrix and its associated Boolean function. Nevertheless, the aforementioned studies are carried out within classical formal contexts.

Based on the Lukasiewicz implication, Elloumi et al. [9] formulated a multi-level conceptual data reduction approach via the reduction of the object sets by keeping only the minimal rows in a formal fuzzy context. Belohlavek et al. [3] proposed a method to reduce the number of formal fuzzy concepts by keeping the so-called crisply generated fuzzy concepts derived from some crisp subset of attributes and leaving out non-crisply generated fuzzy concepts. Li and Zhang [27] introduced the notion of δ-reducts in formal fuzzy contexts, and gave some equivalent characterizations of the δ-consistent sets to determine δ-reducts. Comparing with the studies on knowledge reduction in the classical formal contexts, very little effort has been made to investigate the issue within formal fuzzy contexts. In a concept lattice, object concepts are actually more important, since every formal concept in a concept lattice can be represented as a join of some object concepts. Wu et al. [50] studied granular reducts in classical formal contexts by keeping the extensions of all object concepts. However, it should be noted that the number of formal concepts in a formal fuzzy context dramatically increases, making the structure of the corresponding lattice more complicated than those of a classical formal context. Thus, the reductions made in formal fuzzy contexts become more meaningful.

In this paper, we study granular reducts of formal fuzzy contexts, a generalization of those [50] in fuzzy framework. Accordingly, we propose some granular reduct approaches and investigated the relation between granular reducts and classification reducts in a formal fuzzy context. Specifically, we review in the next section some basic notions and properties of crisp-fuzzy concepts, and then analyze the basic structures of information granules and concept lattices derived from a formal fuzzy context and its sub-contexts. Furthermore, we present some theorems for judging join-irreducible elements in a concept lattice constructed from crisp-fuzzy concepts. In Section 3, we study the issue of granular reducts in formal fuzzy contexts. In Section 4, we propose some granular reduct approaches in consistent formal fuzzy decision contexts. The relationship between granular reduct and classification reduct in a formal fuzzy context is investigated in Section 5. The paper is then concluded with a summary and outlook for further research.

2. Preliminaries

In this section, we recall the notion of a fuzzy concept lattice constructed from crisp-fuzzy concepts and some of its main properties. More details can be found in [53] on crisp-fuzzy concepts.

2.1. Formal fuzzy contexts and crisp-fuzzy formal concepts

Yahia [53] and Krajči [19] independently proposed the “crisp-fuzzy concept”. In the following, we introduce its basic notion and investigate some of its properties used in our subsequent discussion.

Let U be a finite and nonempty set called the universe of discourse. We denote by $\mathcal{P}(U)$ and $\mathcal{F}(U)$ the set of all ordinary subsets of U and the set of all fuzzy sets in universe U, respectively.

For any $\widetilde{X}_1, \widetilde{X}_2 \in \mathcal{F}(U)$, $\widetilde{X}_1 \subseteq \widetilde{X}_2$ if and only if $\widetilde{X}_1(x) \leq \widetilde{X}_2(x)$ $(\forall\ x \in U)$, and operations $\cup$ and $\cap$ on $\mathcal{F}(U)$ are defined by:

$$(\widetilde{X}_1 \cup \widetilde{X}_2)(x) = \widetilde{X}_1(x) \vee \widetilde{X}_2(x),$$
$$(\widetilde{X}_1 \cap \widetilde{X}_2)(x) = \widetilde{X}_1(x) \wedge \widetilde{X}_2(x).$$

The basic data set of FCA is a formal context. A formal fuzzy context is a triple $(U, A, \widetilde{I})$, where U and A are the object set and attribute set respectively, and $\widetilde{I} \in \mathcal{F}(U \times A)$ is a binary fuzzy relation between U and A.

Definition 1 [53]**.** Let $(U, A, \widetilde{I})$ be a formal fuzzy context. For $X \in \mathcal{P}(U)$, $\widetilde{B} \in \mathcal{F}(A)$, the operators $f : \mathcal{P}(U) \to \mathcal{F}(A)$ and $g : \mathcal{F}(A) \to \mathcal{P}(U)$ are defined respectively as follows:

$$f(X)(a) = \bigwedge_{x\in X} \widetilde{I}(x, a),\ \ a \in A,$$
$$g(\widetilde{B}) = \{x \in U | \forall\ a \in A, \widetilde{B}(a) \leq \widetilde{I}(x, a)\}. \tag{1}$$

For any $x \in U$, for simplicity, we will write $f(x)$ instead of $f(\{x\})$.

Operators f and g form a Galois connection between $\mathcal{P}(U)$ and $\mathcal{F}(A)$, and the following properties can be obtained.

Property 1 [53]**.** *Let $(U, A, \widetilde{I})$ be a formal fuzzy context, $X, X_1, X_2, X_i \in \mathcal{P}(U)$, and $\widetilde{B}, \widetilde{B}_1, \widetilde{B}_2, \widetilde{B}_i \in \mathcal{F}(A)$, $i \in J$ (J is an index set). Then*

(1) $X_1 \subseteq X_2 \Rightarrow f(X_2) \subseteq f(X_1)$, $\widetilde{B}_1 \subseteq \widetilde{B}_2 \Rightarrow g(\widetilde{B}_2) \subseteq g(\widetilde{B}_1)$;
(2) $X \subseteq g \circ f(X)$, $\widetilde{B} \subseteq f \circ g(\widetilde{B})$;
(3) $f(X) = f \circ g \circ f(X)$, $g(\widetilde{B}) = g \circ f \circ g(\widetilde{B})$;
(4) $f(\bigcup_{i\in J} X_i) = \bigcap_{i\in J} f(X_i)$, $g(\bigcup_{i\in J} \widetilde{B}_i) = \bigcap_{i\in J} g(\widetilde{B}_i)$.

For a formal fuzzy context $(U, A, \widetilde{I})$, a pair $(X, \widetilde{B}) \in \mathcal{P}(U) \times \mathcal{F}(A)$ satisfying $X = g(\widetilde{B})$ and $\widetilde{B} = f(X)$ is called a crisp-fuzzy concept of $(U, A, \widetilde{I})$ (see [53]). For a set of objects $X \in \mathcal{P}(U)$ and a fuzzy set of attributes $\widetilde{B} \in \mathcal{F}(A)$, from Property 1 (3), we can observe that both $(g\circ f(X), f(X))$ and $(g(\widetilde{B}), f \circ g(\widetilde{B}))$ are crisp-fuzzy concepts. In particular, $(g\circ f(x), f(x))$ is a crisp-fuzzy concept for each $x \in U$ and is called an object concept. For two crisp-fuzzy concepts $(X_1, \widetilde{B}_1)$ and $(X_2, \widetilde{B}_2)$, we define $(X_1, \widetilde{B}_1) \leq (X_2, \widetilde{B}_2)$ if and only if $X_1 \subseteq X_2$ (or equivalently, $\widetilde{B}_2 \subseteq \widetilde{B}_1$). All crisp-fuzzy concepts of $(U, A, \widetilde{I})$ form a complete lattice, denoted as $L(U, \widetilde{A}, \widetilde{I})$, in which the infimum and the supremum are defined respectively as follows:

$$(X_1, \widetilde{B}_1) \wedge (X_2, \widetilde{B}_2) = (X_1 \cap X_2, f \circ g(\widetilde{B}_1 \cup \widetilde{B}_2))$$
$$= (X_1 \cap X_2, f(X_1 \cap X_2));$$
$$(X_1, \widetilde{B}_1) \vee (X_2, \widetilde{B}_2) = (g \circ f(X_1 \cup X_2), \widetilde{B}_1 \cap \widetilde{B}_2)$$