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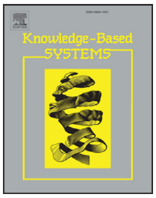

A novel locust swarm algorithm for the joint replenishment problem considering multiple discounts simultaneously

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ABSTRACT

In *B2C E-Commerce* operations, multiple quantity discount offers are commonly practiced in the multi-item replenishment environment. In this paper, a novel joint replenishment model (*JRP*) is presented considering two quantity discounts, all-unit quantity discount, incremental quantity discount, simultaneously. A novel swarms search technique, locust swarms algorithm (*LS*) is introduced and redesigned to solve the novel formulated *JRP* model. Numerical experiments and parameter sensitivity analyses reveal that *LS* is an effective and efficient algorithm for solving the proposed model in terms of solution quality and searching stableness comparing to some other meta-heuristic algorithms, such as *GA*, *DE* and *PSO*. Moreover, management insights such as the mutual effects of multiple quantity discounts to the total cost, and the role of multiple quantity discounts to different stakeholders in replenishment are outlined.

1. Introduction

For most of the item replenishment operations, the motivations for buyers and suppliers to adopt the joint replenishment policy can attribute to two aspects: to pursue the savings in ordering cost [1] and to acquire the discounted order quantities [2]. For example, due to the fast development of *B2C E-Commerce* in China, many corporations of this kind, taking Jingdong, Amazon and Suning for instance, experience unprecedented prosperities in recent years and take up a large share of the market cake, which was once owned by traditional sectors. However, behind the great gains obtained, *B2C* companies often fall into the dilemmas on deciding a best way to group more items in one batch during the replenishment to save ordering cost, and decrease per unit purchasing cost through acquiring the 'coupons' in each order. *B2C* companies are always obsessed by this plight, which inspires them to explore/experiment better strategies to decrease risks in replenishment operations, which is also the original motive that pushes us to lead this research.

In the multi-item system, great benefits can be obtained through grouping multiple items in one order [3]. Traditionally, the joint replenishment problem (*JRP*) is just presented for coping with such kinds of problems, and generally defined as the multi-item inventory problem of coordinating the replenishment of a group of items that may be jointly ordered from a single supplier [4]. The original *JRP* assumes two types of ordering cost, the major ordering cost, which is independent of items in an order and can be taken as the fixed cost, e.g. the set-up cost of the supplier for manufacturing, and the minor ordering cost, e.g. the labor cost and insurance fees, which is closely correlated to the specific item in an order and assumed as the variable cost. However, the superiority of *JRP* is not fully reflected in classic *JRP* models, as group buying different items can not only realize ordering cost savings, but also can acquire discount benefits in replenishment operations.

By investigating the state of the art of *JRP* research, the variations of *JRP* are mainly concentrated on the supplying end and the selling end of a specific supply chain [5]. For example, Hsu et al. [6] assumes satellite factories offering raw materials in the supplying end. Cha et al. [7] consolidates the *JRP* model with delivery process among multiple retailers considered at the selling end. Currently, the research emphases of *JRP* can be generalized to two classifications, *JRP* model-emphasized research and model solving approach-emphasized research [5]. For example, in the former classification, Nielsen and Larsen [8] introduces $Q(s, S)$ inventory policy to *JRP*, Chen and Chen [9] embed channel coordination

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procedures into *JRP*, Moon et al. [10] and Wang et al. [11] investigate *JRP* with delivery considerations. In the latter classification, Kaspi and Rosenblatt [12], Porras and Dekker [13], Zhang et al. [14] and other researchers provide heuristic algorithms for solving *JRP*. While as the *NP-hard* nature of *JRP* has been testified [15], some researchers, like Chan et al. [16], Wang et al. [11,17] and Cui et al. [18], try to present evolutionary algorithms to solve the classical *JRP* model and its extensions.

Of all *JRP* research, one type of *JRP*s with discount consideration has drawn some researchers' attention. However, comparing to various discounting schemes offered by the suppliers in practice and discussed by researchers [19], only a handful of researches have focused on *JRP*s with discount consideration. For example, Chakravarty [20] set up a single retailer *JRP* model total volume discount. Cha and Moon [21] present a *JRP* model with all-unit quantity discount and an efficient heuristic algorithm. Moon et al. [10] construct a *JRP* model with all-unit quantity discount under multiple-supplier environment. Sari et al. [22] consider the situation that the supplier offers time-based quantity discount and construct a joint economic *JRP* model considering the imperfect items and all-unit quantity discount, simultaneously. By addressing the characters of these research models with discounts, however, the corresponding assumptions of these papers are either on a single product replenishing/distribution among several different suppliers or retailers [10,23,24], or on multi-item problems with one type of discount [25–27], no one research takes into consideration of the multiple discounts within the multiple items environment.

In reality, it is a common practice that providing multiple-discount offers considering the supplying ability of specific items in replenishment operations. The inherence of performing discount schemes may stem from the fact that it encourages buying larger batches of items [2], but the presence of different discount schemes often complicates the item purchasing operations [28]. Since different discount schemes, such as the all-unit quantity discount and incremental quantity discount, have different structures, it makes the model even more trivial by integrating discount parameters in one model [2,29]. Conventionally, strategies for grouping items are classified as the direct grouping strategy (*DGS*) and the indirect grouping strategy (*IGS*) [4], but Olsen [[30]] and Wang et al. [31] note that *IGS* outperforms *DGS*. Since *JRP* has been testified as a *NP-hard* problem [15], it is rather challenging for finding an efficient and effective algorithm to solve *JRP* and its variation models [11]. Current approaches for *JRP*s are categorized as heuristics and meta-heuristics, but shortages of these algorithms are reflected in searching universality inefficiently or searching in-depth ineffectively. Henceforth, it is essential to find a proper algorithm for solving the proposed *JRP* model.

Locust Swarms (*LS*) algorithm is extended from particle swarm optimization (*PSO*) that simulates flocks of birds and schools of fish [32] but enhanced the fine searching ability of *PSO* by continuing to search the neighborhood of the current best-found result obtained from *PSO*. Like other probability based algorithms, such as *PSO* [33], artificial bee colony (*ABC*) algorithm [34] and biogeography-based optimization (*BBO*) algorithm [35], *LS* is also a new multi-swarm system and does not make any assumptions about the problem, but has been strengthened the fine searching ability comparing to *PSO* [36], *ABC* [37], *BBO* [38] and other swarm mimic algorithms [39,40]. *LS* is originally designed for multi-modal problems [41,42], the superior performance of which algorithm has been vigorously tested by common benchmark functions [43]. However, its searching abilities in discrete spaces are still unclear, and it has not been applied to solve some more specific problems with integer variables, like *JRP*. The decision variables of *JRP* are discretely distributed among the whole solution spaces, since *LS* featured with the balance of exploration and exploitation in searching abilities, it might be one of the most promising algorithms for solving the *JRP* model with discount considerations. In the following contents, two discount schemes, all-unit discount quantity and incremental discount quantity, are considered simultaneously to construct a new *JRP* model, but the *LS* algorithm should be redesigned to solve the proposed *JRP* model. The main contributions addressed in this research are shown as below.

(1) A novel *JRP* model is developed considering two quantity discounts, the all-unit quantity discount and the incremental quantity discount, simultaneously. In the proposed model, the decision variables are inherited from those in the conventional *JRP*s, namely, the basic cycle time of all items and ordering frequencies of each item. The objective of this new model is to investigate the mutual effects of two discount schemes on the minimized total cost, through which the basic cycle time and ordering frequencies of each item are obtained.
(2) A locust swarms algorithm (*LS*) is introduced and redesigned to solve the proposed model. Considering the structure of the proposed *JRP* model, *LS* is redesigned in its encoding mechanism considering both the integer and real number decision variables, and the searching schemes for generating 'smart' locusts and velocities to update current best points.
(3) Based on the results obtained through numerical experiments, superiorities of *LS* in searching solutions have been testified by a typical case comparing to other meta-heuristic algorithms, such as *GA*, *DE* and *PSO*. Parameter sensitivity analyses of the new *JRP* have been led and the mutual effects of different discounts to the total cost have been testified. Further experiments of *LS* on random parameters of *JRP* and five large scale *JRP* cases confirm the robustness, efficiency and effectiveness of *LS* comparing to *PSO*.

The remainder of this paper is organized as follows. In Section 2, assumptions, notations and the *JRP* model formulation are presented. Section 3 introduces *LS* and its revisions for the proposed *JRP*. Computational results of numerical cases are presented in Section 4. Section 5, conclusions and directions for future research are provided.

2. *JRP* model with two quantity discounts

2.1. Problem description, assumptions and notations

The assumptions of the new model are inherited from those of the conventional *JRP* model, for example, the demand is assumed to be deterministic and conforms to the uniform distribution, no shortages are allowed, no quantity discount, linear holding cost, and so on [4]. The assumptions considered throughout this paper are given below:

The demand of each item is deterministic and constant.
Shortages are not allowed.
The items are replenished when the inventory level drops to zero.
The inventory holding cost is known and constant.
The discount structures are offered by the supplier and known by buyer.
Each type of items is offered one and only one possible discount scheme

The notations of the proposed model are given as follows:

i	the index of items, and set $I = \{i \mid i = 1, 2, \ldots, n\}$
j	the index of discount intervals, and set $J = \{j \mid j = 1, 2, \ldots, J_i\}$

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