Applied Soft Computing 46 (2016) 37–59

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

Deriving the priority weights from incomplete hesitant fuzzy preference relations based on multiplicative consistency

Zhiming Zhang

College of Mathematics and Information Science, Hebei University, Baoding, Hebei 071002, China

ARTICLE INFO

Article history:
Received 30 September 2014
Received in revised form 9 April 2016
Accepted 11 April 2016
Available online 3 May 2016

Keywords:
Hesitant fuzzy set
Hesitant fuzzy preference relation
Incomplete hesitant fuzzy preference relation
Priority weights
Multiplicative consistency

ABSTRACT

In this paper, we define the concept of incomplete hesitant fuzzy preference relations to deal with the cases where the decision makers express their judgments by using hesitant fuzzy preference relations with incomplete information, and investigate the consistency of the incomplete hesitant fuzzy preference relations and obtain the reliable priority weights. We first establish a goal programming model for deriving the priority weights from incomplete hesitant fuzzy preference relations based the α -normalization. Then, we give the definition of multiplicative consistent incomplete hesitant fuzzy preference relations based on the β-normalization, and develop a method for complementing the acceptable incomplete hesitant fuzzy preference relations by using the multiplicative consistency property. Furthermore, utilizing a convex combination method, a new algorithm for obtaining the priority weights from complete or incomplete hesitant fuzzy preference relations is presented on the basis of the β-normalization. Finally, several numerical examples are provided to illustrate the validity and practicality of the proposed methods.

1. Introduction

As an extension of Zadeh's fuzzy sets [46], Torra [26] introduced the concept of hesitant fuzzy sets (HFSs) to enhance the modeling abilities of Zadeh's fuzzy sets. Although the memberships of the elements in a HFS could be any subset of [0, 1], practical works dealing with hesitant fuzzy sets frequently restrict to finite sets [30,43]. To address this issue, Bedregal et al. [8] introduced the notion of typical hesitant fuzzy sets (THFSs). The core of a typical hesitant fuzzy set is typical hesitant fuzzy element (THFE) [8,9], which is composed of several possible values for the membership. THFEs are a very useful tool to express a decision maker (DM)'s hesitancy in providing the preference information over objects in the process of decision making. For example, suppose that a group of decision makers (DMs) are hesitant about some possible values as 0.2, 0.3, and 0.4 to assess the membership of an element x to the set A, and the group of DMs cannot persuade one another to change their own opinions. In such cases, the membership of x to A can be modeled by a THFE represented by $h = \{0.2, 0.3, 0.4\}$, which is significantly different from the situations of using Zadeh's fuzzy sets and its extensions, including interval-valued fuzzy sets [47], intuitionistic fuzzy sets [6], interval-valued intuitionistic fuzzy sets [7], type-2 fuzzy sets [12,44], and fuzzy multisets [20]. Owing to the advantages of handing imprecision whereby two or more sources of vagueness appear simultaneously [50,51].

Decision making is one of the most common activities in the real world. In the process of decision making, an expert (or decision maker) is usually asked to give his/her preferences by comparing the relation of each pair of the considered objects (or alternatives) [39]. Preference relations (or called pairwise comparison matrices, judgment matrices) are very efficient and common tools to express decision makers' preference information on alternatives or criteria [39]. Over the last few decades, a number of studies have focused on the use of preference relations, and various types of preference relations have been developed, including multiplicative preference relation [23], incomplete multiplicative preference relation [14], interval multiplicative preference relation [24], incomplete interval multiplicative preference relation [37], triangular fuzzy multiplicative preference relation [29], incomplete triangular fuzzy multiplicative preference relation [37], fuzzy preference relation [21], incomplete fuzzy preference relation [2], interval fuzzy preference relation [34], incomplete interval fuzzy preference relation [37], triangular fuzzy preference relation [33], incomplete triangular fuzzy preference relation [37], linguistic preference relation [15,16], incomplete linguistic preference relation [35,36], intuitionistic preference relation [38], incomplete intuitionistic preference relation [40], intuitionistic multiplicative preference relation [31], etc. Two important research topics on preference relations are to check their

E-mail addresses: zhimingzhang@ymail.com, zhangzm_1978@126.com

http://dx.doi.org/10.1016/j.asoc.2016.04.010
1568-4946/

consistency and to generate weights from them. Porcel and Herrera-Viedma [22] dealt with incomplete information in a fuzzy linguistic recommender system to disseminate information in university digital libraries. Alonso et al. [1] investigated group decision-making with incomplete fuzzy linguistic preference relations. Herrera-Viedma et al. [17] presented a group decision-making model with incomplete fuzzy preference relations based on additive consistency. Alonso et al. [3] proposed a consistency-based procedure to estimate missing pairwise preference values. Alonso et al. [5] developed a web based consensus support system for group decision making problems and incomplete preferences.

However, the DM may not estimate his/her preference with single exact numerical values, interval numbers, or intuitionistic fuzzy numbers, but with THFEs due to the fact that the DMs are hesitant about some possible values for the preference degrees over paired comparisons of alternatives. In such situations, a hesitant fuzzy preference relation (HFPR), initially proposed by Zhu and Xu [49] on the basis of HFSs, may be more suitable for expressing the DM's hesitant preference information than all the aforementioned preference relations, which do not consider the hesitant fuzzy information and cannot provide all the possible evaluation values of the decision makers when comparing paired alternatives (or criteria). Zhu and Xu [49] proposed a regression method to transform hesitant fuzzy preference relations (HFPRs) into fuzzy preference relations (FPRs). Moreover, Zhu et al. [51] explored the ranking methods with HFPRs in the group decision making environments. Liao et al. [19] investigated the multiplicative consistency of HFPRs and its application in group decision making.

However, it is noted that the aforesaid researches [19,49,51] focused on HFPRs with complete information. A complete HFPR of order n necessitates the completion of all $n(n-1)/2$ judgments in its entire top triangular portion. Sometimes, however, a DM may develop an incomplete HFPR in which some of the elements cannot be provided due to a variety of reasons such as time pressure, lack of knowledge, and the DM's limited expertise related with the problem domain. Consider that it is an interesting and important issue to investigate the consistency of the preference relations and obtain the reliable priority weights, and up to now, no investigation has been devoted to the issue on the approach of deriving the priority vector of incomplete HFPR in the existing literatures. Therefore, it is necessary and significant to pay attention to this issue. In addition, many decision making processes, in the real world, take place in multi-person settings because the increasing complexity and uncertainty of the socio-economic environment makes it less and less possible for single decision maker to consider all relevant aspects of a decision making problem, and the existing researches [19,49,51] only consider an individual HFPR and do not consider group decision making situations. Up to now, considering that no technique has been investigated for dealing with group decision making with incomplete HFPRs. Therefore, this paper introduces the use of a new type of incomplete preference relations, which was pointed out in Ref. [28], as a new challenge to study. We shall in this paper define the concept of incomplete HFPRs, and then construct a goal programming model for deriving the priority weights from incomplete HFPRs under group decision making based on the α-normalization. Then, on the basis of the β-normalization, we define the multiplicative consistent HFPRs and multiplicative consistent incomplete HFPRs, and from which, we propose a method to obtain priority interval weights from a complete or incomplete HFPR. Moreover, a new algorithm is also developed to obtain the collective priority weight vector of several complete or incomplete HFPRs under group decision making situations, and finally, we give several numerical examples to illustrate the proposed model and algorithms.

This paper is structured as follows. Section 2 introduces some known results of FPRs HFSs, and HFPRs. In Section 3, we develop a goal programming model for deriving the priority weights from incomplete HFPRs under group decision making based on the α-normalization. On the basis of the β-normalization, Section 4 defines the multiplicative consistent HFPRs and the multiplicative consistent incomplete HFPRs, based on which, an algorithm is shown to complement an acceptable incomplete HFPR, and a novel procedure is further given to obtain a priority vector from an complete or incomplete HFPR. Moreover, Section 4 also addresses a multiplicative consistency analysis of the collective HFPRs. Then, a practical procedure for obtaining a solution of a GDM problem with several complete or incomplete HFPRs is presented. Finally, the main conclusions are given in Section 5.

2. Preliminaries

In this section, we will briefly recall the concepts of fuzzy preference relation, hesitant fuzzy set, and hesitant fuzzy preference relation.

2.1. Fuzzy preference relation

Definition 2.1 *([21])*. Let $X = \{x_1, x_2, \ldots, x_n\}$ be a set of alternatives, then $R = (r_{ij})_{n\times n}$ is called a fuzzy preference relation (FPR) on $X \times X$ with the following conditions:

$$r_{ij} \geq 0, \quad r_{ij} + r_{ji} = 1, \quad i, j = 1, 2, \ldots, n, \tag{1}$$

where r_{ij} denotes the degree that the alternative x_i is prior to the alternative x_j provided by the decision maker. Especially, $r_{ij} = 0.5$ indicates indifference between x_i and x_j; $r_{ij} > 0.5$ indicates x_i is preferred to x_j, the larger the r_{ij}, the greater the preference degree of the alternative x_i over x_j, $r_{ij} = 1$ indicates that x_i is absolutely prior to x_j; $r_{ij} < 0.5$ indicates x_j is preferred to x_i; the smaller the r_{ij}, the greater the preference degree of the alternative x_j over x_i, $r_{ij} = 0$ indicates that x_j is absolutely prior to x_i.

Definition 2.2 *([25])*. Let $R = (r_{ij})_{n\times n}$ be a FPR, then $R = (r_{ij})_{n\times n}$ is called a multiplicative consistent FPR if it satisfies the multiplicative transitivity property:

$$r_{ik}r_{kj}r_{ji} = r_{ki}r_{jk}r_{ij}, \quad i, j, k = 1, 2, \ldots, n, \tag{2}$$

By the simple algebraic manipulation, Eq. (2) can be expressed as [10,11]

$$r_{ij} = \frac{r_{ik}r_{kj}}{r_{ik}r_{kj} + (1 - r_{ik})(1 - r_{kj})}, \quad i, j, k = 1, 2, \ldots, n \tag{3}$$

where $r_{ik} > 0$, $i, k = 1, 2, \ldots, n$.