Impulsive H_∞ control of discrete-time Markovian jump delay systems

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***Abstract*—This paper considers the impulsive H_∞ control problem for a class of discrete-time Markovian jump delay systems for the first time. Some sufficient conditions are given in terms of matrix inequalities. A numerical example is presented to show the effectiveness of the theoretical results.**

Keywords: Delay; Discrete-time Markovian jump system; H_∞ control; Impulsive control; Lyapunov functional

I. Introduction

As is well known, time delay is commonly encountered in various engineering systems and is frequently a source of instability and poor performance. At the same time, in many real world systems, such as optimal control models in economics, circuit networks, frequency modulated systems and biological systems, the underlying systems are under the influence of time delay, and sometimes contain abrupt changes of their states at certain time moments. Therefore, systems with impulses or impulsive control of systems have attracted the interest of many researchers, see [1–10] and the references therein.

Meanwhile, the structures of the physical plants may experience random abrupt changes, such as stochastic failures and repairs of the components, changes in the interconnections of subsystems, and sudden environment changes. The system with Markovian jump parameters is very appropriate to describe this kind of dynamics. Applications of Markovian jump systems can be found in manufacturing systems, networked control systems, economics systems, air intake system, and other practical systems. Recently, a lot of attention has been paid to the stability analysis and controller synthesis for Markovian jump systems, see [6, 9–13] and the references therein.

When controlling a real plant, it is useful to design a control system which is not only stable but also guarantees an adequate level of performance. The performance indices of the systems, such as the robustness and disturbance rejection attenuation level, have not received enough attention for discrete-time impulsive Markovian jump delay systems despite their importance in practical applications. To the best of the author's knowledge, the impulsive H_∞ control problem for discrete-time Markovian jump delay systems has not been investigated yet. This motivated the investigation of this paper.

In this paper, the impulsive H_∞ control problem is considered for discrete-time time-delay systems with Markovian jump parameters. Firstly, a criterion is given for the considered system to be stochastically stable with an H_∞ performance level. Based on this, a LMI-based approach is proposed to design an impulsive controller such that the resultant system is stochastically stable and satisfies a prescribed H_∞ performance. A numerical example is provided to demonstrate the effectiveness of the proposed results.

The rest of this paper is organized as follows. Section 2 introduces the model, some notations and assumptions. In Section 3, first the stochastic stability and H_∞ noise attenuation performance for the proposed system is investigated, then an impulsive controller is designed. A numerical example and its simulations are given to illustrate the effectiveness of the obtained results in Section 4. Finally, Section 5 draws a general conclusion.

Notations. In the sequel, the standard notations are used. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. Matrices, if not explicitly stated, are assumed to have compatible dimensions. For a matrix X, $X > 0(X \geq 0)$ means that X is symmetric and positive (semipositive) definite, and $\lambda_{\max}(X)$, $\lambda_{\min}(X)$ denote the maximal and minimal eigenvalues of X, respectively. $\mathbb{Z}^+$ denotes the set of positive integers. $\mathbb{N}$ denotes the set of natural numbers, i.e., $\mathbb{N} = \{0, 1, 2, \cdots\}$. $E\{\cdot\}$ stands for the mathematical expectation. I represents the identity matrix. Let $N(l, k)$ denote the impulses on $(l, k]$ and assume $N(k, k) = 0$.

II. Preliminaries

Consider the following discrete-time Markovian jump systems with impulsive control and time delay in a fixed complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$:

$$\begin{cases} x(k+1) = A(r_k)x(k) + A_d(r_k)x(k-d) \\ \qquad\qquad +D(r_k)w(k), & k \neq \tau_j; \\ x(\tau_j+1) = H(r_{\tau_j})x(\tau_j), & j \in \mathbb{Z}^+; \\ z(k) = C(r_k)x(k) + C_d(r_k)x(k-d) \\ \qquad\quad +F(r_k)w(k); \\ x(\theta) = \varphi(\theta), & \theta \in J. \end{cases} \tag{1}$$

where $k \in \mathbb{N}$, $x(k) \in \mathbb{R}^n$ is the state of the system, $z(k) \in \mathbb{R}^{n_1}$ is the controlled output, $w(k) \in \mathbb{R}^{n_2}$ is the disturbance input, which belong to $l_2[0, \infty)$, where $l_2[0, \infty) = \{\{a_k, k \geq 0\}| \sum_{k=0}^{\infty} a_k^2 < \infty\}$. d denotes the time delay of the state, $d \in \mathbb{Z}^+$, $J = \{-d, -d+1, \cdots, -1, 0\}$, $\{r_k, k \in \mathbb{N}\}$ is a discrete-time homogeneous Markov chain with finite state space $\mathbb{S} = \{1, 2, \cdots, s\}$, $\tau_i \in \mathbb{Z}^+ (i \in \mathbb{Z}^+)$ are the impulsive instants, $\tau_1 < \tau_2 < \cdots < \tau_m < \cdots$, $\tau_m \to \infty$ when $m \to \infty$. Let $\tau_0 = 0$. φ is the initial condition of the state. $A(r_k), A_d(r_k), D(r_k)$, $H(r_k), C(r_k)$, $C_d(r_k)$, $F(r_k)$ $(r_k \in \mathbb{S})$ are real matrices.

Denote the state transition matrix by $\Pi = (\pi_{ij})_{i,j \in \mathbb{S}}$, i.e., the transition probabilities of $\{r_k, k \in \mathbb{N}\}$ are given by

$$Pr\{r(k+1) = j | r(k) = i\} = \pi_{ij} \quad \text{for} \quad i, j \in \mathbb{S},$$

with $\pi_{ij} \geq 0$ for $i, j \in \mathbb{S}$ and $\sum_{j=1}^{s} \pi_{ij} = 1$ for $i \in \mathbb{S}$.

Definition 1. *The discrete-time impulsive Markovian jump delay system (1) with $w(k) = 0$ is said to be stochastically stable, if for every initial state (φ, r_0), the following holds:*

$$E\left\{\sum_{k=0}^{\infty} \|x(k;\varphi,r_0)\|^2 | \varphi, r_0\right\} < \infty.$$

Definition 2. *System (1) is said to be stochastically stable with noise attentuation level γ, if the system with $w(k) = 0$ is stochastically stable and the output vector $z(k)$, under zero initial condition, satisfies*

$$\sum_{l=0}^{\infty} E\{z^T(l)z(l)|\varphi, r_0\} \leq \gamma^2 \sum_{l=0}^{\infty} w^T(l)w(l),$$

for any non-zero $w(k) \in l_2[0,\infty)$.

The problem to be addressed in this paper can be formulated as follows: For given $\gamma > 0$, designing an impulsive controller,

$$x(\tau_j + 1) = H(r_{\tau_j})x(\tau_j), j \in \mathbb{Z}^+$$

where $H(r_{\tau_j})$ are constant gains to be designed, such that the resultant system is stochastically stable with noise attentuation level γ.

III. Main results

First of all, we present a theorem for system (1), which will play a key role in solving the controller design problem.

Theorem 1. *For given positive scalars $\mu \geq 1, 0 < c < 1$, $\gamma > 0$, if there exist matrices $P(i) > 0$ $(i = 1, 2, \cdots, s)$, $Q_l > 0 (l = 1, 2, \cdots, d)$, $q \in \mathbb{Z}^+$, such that the following inequalities hold for all $i \in \mathbb{S}$,*

$$\Xi(i) = \begin{pmatrix} \Xi_{11} & \Xi_{12} & A^T(i)\bar{P}(i)D(i) + C^T(i)F(i) \\ \star & \Xi_{22} & A_d^T(i)\bar{P}(i)D(i) + C_d^T(i)F(i) \\ \star & \star & \Xi_{33} \end{pmatrix} < 0, \quad (2)$$

where $\Xi_{11} = -\mu P(i) + Q_1 + A^T(i)\bar{P}(i)A(i) + C^T(i)C(i)$, $\bar{P}(i) = \sum_{j=1}^{s} \pi_{ij}P(j)$, $\Xi_{12} = A^T(i)\bar{P}(i)A_d(i) + C^T(i)C_d(i)$, $\Xi_{22} = -\mu Q_d + A_d^T(i)\bar{P}(i)A_d(i) + C_d^T(i)C_d(i)$, $\Xi_{33} = -\eta I + D^T(i)\bar{P}(i)D(i) + F^T(i)F(i)$, $\eta \leq \gamma^2 c^2 \mu^{-2}$;

$$\tilde{\Omega}(i) = \begin{pmatrix} \tilde{\Omega}_{11} & C^T(i)C_d(i) & C^T(i)F(i) \\ \star & \tilde{\Omega}_{22} & C_d^T(i)F(i) \\ \star & \star & -\eta I + F^T(i)F(i) \end{pmatrix} < 0, \quad (3)$$

where $\tilde{\Omega}_{11} = -cP(i) + Q_1 + H^T(i)\bar{P}(i)H(i) + C^T(i)C(i)$, $\tilde{\Omega}_{22} = -cQ_d + C_d^T(i)C_d(i)$;

$$Q_{l+1} - cQ_l \leq 0, \quad l = 1, 2, \cdots, d-1; \quad (4)$$

$$\tau_j - \tau_{j-1} = q, \quad j \in \mathbb{Z}^+; \quad (5)$$

$$\ln(c\mu^{-1}) + q \ln \mu < 0; \quad (6)$$

then system (1) is stochastically stable with an H_∞ noise attenuation performance index γ.

Proof. Let $\mathcal{X}_k = \{x(k-d), \quad x(k-d+1), \quad \cdots, \quad x(k)\}$. Choose the following Lyapunov functional

$$V(\mathcal{X}_k, r_k) = V_1(\mathcal{X}_k, r_k) + V_2(\mathcal{X}_k, r_k)$$

where $V_1(\mathcal{X}_k, r_k) = x^T(k)P(r_k)x(k)$, $V_2(\mathcal{X}_k, r_k) = \sum_{l=1}^{d} x^T(k-l)Q_l x(k-l)$.

(1) Firstly, we will prove the stochastic stability of the system (1) with $w(k) = 0$.

When $k \neq \tau_m, m \in \mathbb{Z}^+$, for $r_k = i, r_{k+1} = j$,

$$\begin{aligned} &E\{V_1(\mathcal{X}_{k+1}, r_{k+1})|\mathcal{X}_k, r_k\} - \mu V_1(\mathcal{X}_k, r_k) \\ &= [A(i)x(k) + A_d(i)x(k-d)]^T \bar{P}(i)[A(i)x(k) \\ &\quad + A_d(i)x(k-d)] - \mu x^T(k)P(i)x(k); \end{aligned} \quad (7)$$

$$\begin{aligned} &E\{V_2(\mathcal{X}_{k+1}, r_{k+1})|\mathcal{X}_k, r_k\} - \mu V_2(\mathcal{X}_k, r_k) \\ &= \sum_{l=1}^{d} x^T(k+1-l)Q_l x(k+1-l) \\ &\quad -\mu \sum_{l=1}^{d} x^T(k-l)Q_l x(k-l) \\ &= x^T(k)Q_1 x(k) + \sum_{l=2}^{d} x^T(k+1-l)Q_l x(k+1-l) \\ &\quad -\mu \sum_{l=1}^{d-1} x^T(k-l)Q_l x(k-l) - \mu x^T(k-d)Q_d x(k-d) \\ &= \sum_{l=1}^{d-1} x^T(k-l)[Q_{l+1} - \mu Q_l]x(k-l) \\ &\quad -\mu x^T(k-d)Q_d x(k-d) + x^T(k)Q_1 x(k). \end{aligned} \quad (8)$$

From (4) ($Q_{l+1} - cQ_l \leq 0$, $l = 1, 2, \cdots, d-1$), $\mu \geq 1, 0 < c < 1$, we have $Q_{l+1} - \mu Q_l \leq 0$, $l = 1, 2, \cdots, d-1$. Therefore, from (7), (8), when $k \neq \tau_m, m \in \mathbb{Z}^+$, for $r_k = i, r_{k+1} = j$,

$$E\{V(\mathcal{X}_{k+1}, r_{k+1})|\mathcal{X}_k, r_k\} - \mu V(\mathcal{X}_k, r_k) \leq \xi^T(k)\Upsilon(i)\xi(k),$$

where $\xi(k) = \begin{pmatrix} x^T(k) & x^T(k-d) \end{pmatrix}^T$, $\Upsilon(i) = \begin{pmatrix} -\mu P(i) + Q_1 + A^T(i)\bar{P}(i)A(i) & A^T(i)\bar{P}(i)A_d(i) \\ \star & -\mu Q_d + A_d^T(i)\bar{P}(i)A_d(i) \end{pmatrix}$.

From (2), $\Xi(i) < 0$ can be rewritten as

$$\begin{pmatrix} \Upsilon(i) & \Theta \\ \star & -\eta I + D^T(i)\bar{P}(i)D(i) \end{pmatrix} + \begin{pmatrix} C^T(i) \\ C_d^T(i) \\ F^T(i) \end{pmatrix} \begin{pmatrix} C(i) & C_d(i) & F(i) \end{pmatrix} < 0,$$

where $\Theta = \begin{pmatrix} A^T(i)\bar{P}(i)D(i) \\ A_d^T(i)\bar{P}(i)D(i) \end{pmatrix}$, which implies $\begin{pmatrix} \Upsilon(i) & \Theta \\ \star & -\eta I + D^T(i)\bar{P}(i)D(i) \end{pmatrix} < 0$, and thus $\Upsilon(i) < 0$.

Thus, for $r_k = i, r_{k+1} = j$,

$$\begin{aligned} &E\{V(\mathcal{X}_{k+1}, r_{k+1})|\mathcal{X}_k, r_k\} - \mu V(\mathcal{X}_k, r_k) \\ &\leq \xi^T(k)\Upsilon(i)\xi(k) \leq 0, \quad k \neq \tau_m, m \in \mathbb{Z}^+. \end{aligned} \quad (9)$$

When $k = \tau_m, m \in \mathbb{Z}^+$, for $r_{\tau_m} = i$, $r_{\tau_m+1} = j$,

$$\begin{aligned} &E\{V_1(\mathcal{X}_{k+1}, r_{k+1})|\mathcal{X}_k, r_k\} - cV_1(\mathcal{X}_k, r_k) \\ &= x^T(k)H^T(i)\bar{P}(i)H(i)x(k) - cx^T(k)P(i)x(k); \end{aligned} \quad (10)$$

$$\begin{aligned} &E\{V_2(\mathcal{X}_{k+1}, r_{k+1})|\mathcal{X}_k, r_k\} - cV_2(\mathcal{X}_k, r_k) \\ &= \sum_{l=1}^{d-1} x^T(k-l)[Q_{l+1} - cQ_l]x(k-l) \\ &\quad -cx^T(k-d)Q_d x(k-d) + x^T(k)Q_1 x(k). \end{aligned} \quad (11)$$