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Periodicity and stability for variable-time impulsive neural networks

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ABSTRACT

The paper considers a general neural networks model with variable-time impulses. It is shown that each solution of the system intersects with every discontinuous surface exactly once via several new well-proposed assumptions. Moreover, based on the comparison principle, this paper shows that neural networks with variable-time impulse can be reduced to the corresponding neural network with fixed-time impulses under well-selected conditions. Meanwhile, the fixed-time impulsive systems can be regarded as the comparison system of the variable-time impulsive neural networks. Furthermore, a series of sufficient criteria are derived to ensure the existence and global exponential stability of periodic solution of variable-time impulsive neural networks, and to illustrate the same stability properties between variable-time impulsive neural networks and the fixed-time ones. The new criteria are established by applying Schaefer's fixed point theorem combined with the use of inequality technique. Finally, a numerical example is presented to show the effectiveness of the proposed results.

1. Introduction

It is well-known that the theory of impulsive neural networks is not only richer than the corresponding theory of neural networks but also successfully applies to many kinds of research areas (Chen, Luo, & Zheng, 2016; He, Qian, & Cao, 2017; Zhou & Zhang, 2016; Zhu & Cao, 2012a, b; Zhu, Cao, & Rakkiyappan, 2015), such as automatic control, secure communication, image encryption, pattern recognition, associative memory and modeling natural phenomena. Many interesting results for stability and synchronization of impulsive neural networks have been obtained (He, Qian, & Lam, 2015; Liu, Liu, & Xie, 2011; Song, Yan, & Zhao, 2016; Wang, Li, & Huang, 2014; Yang, Guo, & Wang, 2016; Yang & Xu, 2005; Zhang, Li, & Huang, 2014; Zhu, 2014; Zhu & Cao, 2010; Zhu, Rakkiyappan, & Chandrasekar, 2014; Zhu & Song, 2011). In Zhu and Song (2011), authors studied the exponential stability of impulsive nonlinear stochastic differential equations with mixed time delays. Yang, Guo et al. (2016) and Zhang et al. (2014) investigated global synchronization of multiple recurrent neural networks with time delays via impulsive interactions. In Zhu et al. (2014), Zhu et al. developed the idea originated with Lyapunov–Krasovskii functional to impulsive bidirectional associative memory neural networks and established some global exponential stability criteria of impulsive BAM neural networks with both Markovian jump parameters and time delays. However, in real world problems, the impulse of many systems do not occur at fixed time (Liu & Ballinger, 2002; Liu, Li, & Liao, 2011; Liu, Liu, & Yang, 2016; Liu & Wang, 2006; Nie, Teng, & Hu, 2010; Nie, Teng, & Lin, 2009; Şaylı & Yılmax, 2014; Stamova & Kergomard, 2016; Yang, Li, & Song, 2016). For example, in population control systems (Nie et al., 2010), in some circuit control systems (Liu, Li et al., 2011; Liu et al., 2016; Şaylı & Yılmax, 2014), and in some ecological systems (Nie et al., 2009), the ecological system is often affected by environmental changes and human activities, these short-time perturbations are often assumed to be in the form of impulses in the modeling process. These work assume the impulsive effects have the same threshold, i.e., at the same time. But they ignored the side effects of pesticide on natural enemies, it is unreasonable that they assumed the time of spraying pesticide and releasing natural enemies is the same. Therefore, some systems with variable-time impulses have more practical value. Moreover, system with variable-time impulsive is more general than the fixed-time impulses.

Based on different characteristics of impulsive events (Akhmet, 2010; Lakshmikantham, Bainov, & Simeonov, 1989; Rachunková & Tomeček, 2015), we usually encounter three primary types of impulsive systems: in which impulses occur at fixed time; in which impulses occur when the trajectory hits a hypersurface in the extended phase space; which are discontinuous dynamic systems. The impulse moments in fixed-time impulsive systems can be prescribed, which in variable-time impulsive systems cannot be prescribed, and not known until one starts to look for a certain solution. The solutions of variable-time impulsive systems starting

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at different initial conditions have different points of discontinuity. The solutions of variable-time impulsive systems may experience the pulse phenomenon or beating phenomenon, namely the solutions may hit the same hypersurface finite or infinite number of times causing rhythmical beating. We need to find desirable conditions that guarantee the absence or presence of pulse phenomena, and unfortunately, some fruitful methods are of limited use for impulsive problems with variable time impulses. Therefore, variable-time impulsive systems are more difficult to study than fixed-time impulsive systems.

Recently, there are a large number of interesting results on existence and stability of periodic solution of fixed-time impulsive neural networks or neural networks in the literature, see Aouiti, M'hamdi, Cao, and Alsaedi (2016), Bao (2016), Li, Bohner, and Wang (2015), Li, Jiang, and Hu (2016), Li and Song (2013, 2016), Pan and Cao (2011), Wang and Agarwal (2016) and Yang, Liao, and Evans (2005) and the references therein. However, it should be pointed out that the existence and stability analysis of periodic solution of variable-time impulsive neural networks was rarely considered (Nie, Teng, & Hu, 2011; Qi & Fu, 2001; Şaylı & Yılmax, 2015, 2016). In Nie et al. (2011), Nie studied the existence and stability of positive order-1 or order-2 periodic solution of a stage-structured model with state-dependent impulsive effects by using poincaré map and the analogue of Poincaré's criterion. In Şaylı and Yılmax (2016), authors investigated the existence and uniqueness of exponentially stable anti-periodic solution for state-dependent impulsive recurrent neural networks by employing method of coincide degree theory combined with an appropriate Lyapunov function. Qi and Fu (2001) investigated the existence of the limit cycles of impulsive differential equations with impulses at variable times. In Şaylı and Yılmax (2015), based on the B-equivalence method, authors reduced state-dependent impulsive neural networks system to an equivalent fixed-time impulsive system. Moreover, by applying the Mawhin's continuation theorem of coincide degree theory, a series of sufficient criteria for the existence and global exponential stability of periodic solution of state-dependent impulsive neural networks system were obtained. Summing up the above, most existing literatures about periodic solution of impulsive neural networks have only focused on the case of fixed-time impulses (Aouiti et al., 2016; Bao, 2016; Li et al., 2015, 2016; Li & Song, 2013, 2016; Pan & Cao, 2011; Wang & Agarwal, 2016; Yang et al., 2005). To the best of our knowledge, there is little work on the stability of periodic solution of neural networks with variable-time impulses at present stage (Nie et al., 2011; Qi & Fu, 2001; Şaylı & Yılmax, 2015, 2016). Moreover, the stability theory of periodic solution of neural networks with variable-time impulses is very important and significant. Consequently, it is necessary to further investigate stability behaviors in an array of periodic solution of neural networks with variable-time impulses.

In the existing literatures (Akhmet, 2010; Lakshmikantham et al., 1989; Liu & Ballinger, 2002; Liu & Wang, 2006; Şaylı& Yılmax, 2014, 2015, 2016; Stamova & Kergomard, 2016; Yang, Li et al., 2016), comparison principle is often used for the stability analysis of variable-time impulsive neural networks. In Akhmet (2010), Akhmet proposed a powerful analytical tool for variable-time impulsive systems, i.e., B-equivalence, which was also a method of comparison systems. Based on comparison principle, we can reduce the variable-time impulsive system to a fixed-time impulsive system. The reduced fixed-time impulsive neural networks are considered as the comparison system of variable-time impulsive neural networks. It is also worth noting that the jumping operator in comparison system might be very complex map, and it is difficult to find the jumping operator such as Şaylı and Yılmax (2014, 2015, 2016). In this paper, it is the main difficulty that to estimate the relationship between the original jumping operator in variable-time impulsive neural networks and new jumping operator in reduced fixed-time impulsive neural networks. Inspired by the above discussion, we have to say that it is necessary to propose comparison principle to tackle this problem. Evidently, the proposed methods in this paper are meaningful and can formulate the relationship between original and corresponding new jump operators. We shall extend the comparison system to variable-time impulsive neural networks and establish a new comparison system, which is different from the existing results (Şaylı& Yılmax, 2014, 2015, 2016). By applying comparison principle, the existence and global exponential stability of periodic solution of neural networks with variable-time impulses is considered.

Motivated by the above discussions, the main contribution of this paper lies in the following aspects. Firstly, this paper aims to formulate a throughout theoretical framework of reduction and comparison principle for variable-time impulsive neural networks. Additionally, we shall propose the sufficient conditions that ensure every solution of system intersects each surface of discontinuity exactly once, analyze and formulate the relationship between original and corresponding new jump operation. Subsequently, we shall theoretically prove that the stability of corresponding comparison system guarantee same stability of variable-time impulsive neural networks, which holds for some previous results to some extent. Finally, we shall consider the existence and global exponential stability of periodic solution of variable-time impulsive neural networks by applying comparison principle.

The paper is organized as follows. In Section 2, a class of variable-time impulsive neural networks is introduced, and some necessary assumptions, definitions and lemma are given. In Section 3, we list several assumptions under which each proposed solution of our well-defined models intersects with each discontinuous surface exactly once, furthermore, the original systems will be reduced to the fixed-time impulsive ones by constructing a map. In Section 4, we shall theoretically prove that the existence of periodic solution of the comparison system of the variable-time impulsive system, In Section 5, we show that the global exponential stability of corresponding system with fixed-time impulses imply the same stability property of the variable-time impulsive system. Several sufficient conditions for the uniqueness and global exponential stability of periodic solution of neural networks with variable-time impulses are derived. In Section 6, we give a numerical example with numerical simulations to show the effectiveness of the proposed results. Finally, we draw conclusion in Section 7.

Notations: Let $\mathbb{R}$, $\mathbb{R}_+$, and $\mathbb{Z}$ denote the sets of real numbers, nonnegative real numbers and positive integers, respectively. Moreover, $\mathbb{R}^n$ denotes the n-dimensional real spaces, respectively, equipped with the Euclidean norm $\|\cdot\|$. And we denote $\Gamma_k = \{(t, x(t)) \in \mathbb{R}_+ \times \mathbb{R}^n : t = \theta_k + \tau_k(x(t)), t \in \mathbb{R}_+, x \in \mathbb{R}^n\}$ the kth surface of discontinuity. For a given continuous ω-periodic function $g(t)$ defined on $\mathbb{R}^n$, we define $g^u = \max_{t\in[0,\ \omega]}|g(t)|$. For a given matrix $C(t) = (c_{ij}(t))_{n\times n}$, we define $C^u = \max_{t\in[0,\ \omega]}\|C(t)\|$.

2. Model formulation and preliminaries

Consider the following variable-time impulsive neural networks:

$$\begin{cases} \dfrac{dx_i(t)}{dt} = -a_i(t)x_i(t) + \displaystyle\sum_{j=1}^{n} b_{ij}(t)f_j(x_j(t)) + I_i(t), \\ \triangle x_i(t)|_{t=\theta_k+\tau_k(x_i(t))} = S_{ik}(x_i(t)), \end{cases} \tag{1}$$

for $i = 1, 2, \ldots, n$, where n corresponds to the number of units in a neural network, $x_i(t)$ denotes the state variable associated with the ith neuron, $a_i(t) > 0$ represents the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs at time t, $b_{ij}(t)$ corresponds to the synaptic connection weight of the unit j on