Applied Soft Computing 46 (2016) 246–259

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

A framework for radial data comparison and its application to fingerprint analysis

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ARTICLE INFO

Article history:
Received 5 October 2015
Received in revised form 11 April 2016
Accepted 4 May 2016
Available online 6 May 2016

Keywords:
Radial data
Restricted Equivalence Function
Similarity Measure
Fingerprint singular point detection

ABSTRACT

This work tackles the comparison of radial data, and proposes comparison measures that are further applied to fingerprint analysis. First, we study the similarity of scalar and non-scalar radial data, elaborated on previous works in fuzzy set theory. This study leads to the concepts of restricted radial equivalence function and Radial Similarity Measure, which model the perceived similarity between scalar and vectorial pieces of radial data, respectively. Second, the utility of these functions is tested in the context of fingerprint analysis, and more specifically, in the singular point detection. With this aim, a novel Template-based Singular Point Detection method is proposed, which takes advantage of these functions. Finally, their suitability is tested in different fingerprint databases. Different Similarity Measures are considered to show the flexibility offered by these measures and the behavior of the new method is compared with well-known singular point detection methods.

1. Introduction

The ability to quantify the similarity between two objects in a given universe is a pillar in applied fields of research. Historically, this quantification has been based on metrics, which are able to capture, in a sensible (and coherent) manner, the proximity of any two objects in a measurable universe. Metrics hold very interesting properties, specifically triangular inequality, which preserves the notion that the shortest path between two objects is the straight one. However, they also impose the need for the representation of the objects in a metric space, as well as notions (e.g., transitivity), which are not natural in certain scenarios [1].

When it comes to measuring dissimilarity between multivalued data, L_p metrics often come as a straightforward option; the most relevant case is $p=2$, which recovers the Euclidean metric. The L_p metric has been long criticized, specially regarding its low accuracy in capturing perceptual dissimilarities. For example, Attneave stated that the assumption that the *psychological space is Euclidean in its character* is *exceedingly precarious* [2]. Obviously, there exist other metrics yielding more (perceptually) accurate measurements of dissimilarity, specially when they are designed for well-defined scenarios [3,4]. The debate about the restrictivity of the requisites imposed by metrics is still open [5]. Literature contains both practical [6], and theoretical criticisms. Authors as Tversky [7] or Santini and Jain [5] criticized the necessity of imposing metric conditions to Similarity Measures, as well as the representation of objects in metric spaces, given that they are often missing in human understanding. Tversky [7,8] also revisited the necessity of symmetry and the directional nature of comparisons in certain scenarios. Finally, the low representativity of the values given by metrics for large-range comparisons has also been under debate [9,10].

Different mathematical theories have tackled the modelling of similarity with tools other than metrics, leading to what Zadeh referred to as *a vast armamentarium* of techniques for comparison [11]. In fact, even axiomatic representations of non-metric comparison frameworks have appeared in the literature (e.g., [7] for set-based similarity, or [12,13] for T-indistinguishability). In the context of fuzzy set theory, a range of authors have elaborated on the semantic interpretation of similarities and dissimilarities [5], since Zadeh introduced similarity as an extension of equivalence [11]. This is natural, considering that the concepts of proximity and similarity (as well as ordering or clustering) are strongly related to human interpretation, and hence prone to be tackled in fuzzy

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http://dx.doi.org/10.1016/j.asoc.2016.05.003
1568-4946/

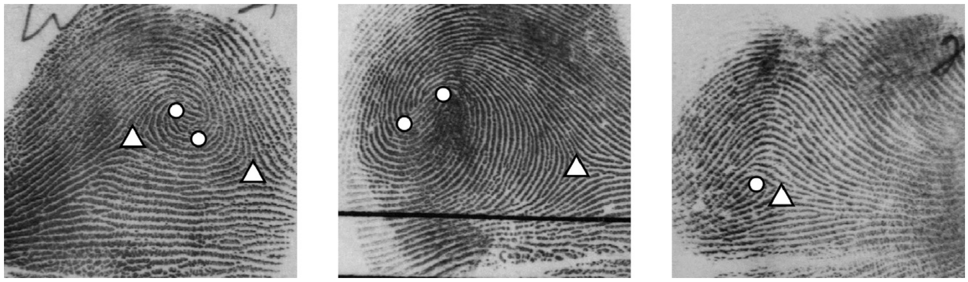

Fig. 1. Examples of singular points detected on fingerprints extracted from the NIST-4 dataset [19]. The *deltas*, represented as triangles, are triangular-like ridge confluences, while *cores*, represented as circles, take place at curly ridge structures.

terms. A large variety of proposals have appeared for modelling both similarity and dissimilarity; in this work we focus our interest on two of them: Restricted Equivalence Functions (REFs) for the comparison of membership degrees and Similarity Measures (SMs) for the comparison of fuzzy sets on discrete universes [14].

In this paper we propose a definition of the concept of REFs and SMs for radial data. This study is motivated by the increasing relevance of radial data in real applications, especially in those demanding the extraction of information by means of computer vision techniques. Very often, computer vision handles radial data in different flavours (e.g., angular, vector or tensorial data [15]) and consequently demands well-defined operators for different tasks, including data comparison. Typically, the study of radial data has been restricted to radial statistics, which mostly study the fitting and analysis of well-known distributions on radial set-ups. To the best of our knowledge, there are no studies on the quantification of the similarity of elements in a radial universe. This situation has led many researchers to use *ad-hoc* operators to deal with the special conditions of the data, instead of creating a framework in which different operators can be encompassed. For this reason in this work we develop a framework aiming at easing the comparison of radial data. More specifically, we define Restricted Radial Equivalence Functions (RREFs), as well as Restricted Similarity Measures (RSMs), which attempt to mimic the behaviour of REFs and SMs in radial universes.

As a case of study, we present an application of RREFs and RSMs to biometric identification, specifically to singular point detection in fingerprint recognition [16]. Fingerprints can be seen as a set of ridges (lines) that represent the relief of the skin in the fingertip surface. Hence, their analysis is often based on studying the line patterns in a local or semi-local basis. Within fingerprint analysis, a fundamental operation is the detection and localization of the so-called *Singular Points* (SPs), which are structural singularities in the ridges (see Fig. 1). SP detection is often related to specific occurrences in the orientation of the ridges of neighbouring regions, which are usually found using semi-local analysis [17] or complex convolution filters [18].

On this account, a simple yet effective framework for SP detection is presented in this paper by means of RREFs and RSMs, which shows the usefulness and flexibility of these new measures. Furthermore, other well-known SP detection algorithms have been used as a baseline for performance evaluation [20,21]. In this comparative analysis we have considered two different types of databases: NIST-4 database [19], the most commonly used fingerprint database and synthetic fingerprint databases generated by SFinGe.[1]

The remainder of the work is as follows. In Section 2 we review the concepts of REF and SM, as well as some standard notation on radial data. Section 3 is devoted to introduce the concepts of RREF and RSM. Both RREF and RSM are used in Section 4, in which we present our proposal for SP detection in fingerprints. Section 5 includes an experimental study in which we illustrate the performance of our SP detection method, compared to other well-known methods in the literature. Finally, Section 6 gathers some conclusions and a brief discussion on potential future evolutions of our method.

2. Preliminaries

Among the areas in which fuzzy set theory has played a relevant role, data similarity modelling is one of the most prominent. The reason is that the natural concepts of similarity, closeness or likeliness are inherently bounded to human interpretation. Hence, different proposals have appeared to effectively model the comparison of pieces of information. Among these, we find fuzzy metric spaces [6], with interesting advantages over classical metric spaces in terms of interpretability [22] or equivalence and Similarity Measures [14], which we take as inspiration to develop measures that can handle radial data. Next, we recall the concepts of REF and SM.

Definition 1. A continuous, strictly decreasing function $n:[0, 1]\to[0, 1]$ such that $n(0)=1$, $n(1)=0$ and $n(n(x))=x$ for all $x\in[0, 1]$ (involutive property) is called strong negation.

Definition 2. [14] A mapping $r:[0, 1]^2\to[0, 1]$ is said to be a Restricted Equivalence Function (REF) associated with the strong negation n if it satisfies the following:

(R1) $r(x, y)=r(y, x)$ for all $x, y\in[0, 1]$;
(R2) $r(x, y)=1$ if and only if $x=y$;
(R3) $r(x, y)=0$ if and only if $\{x, y\}=\{0, 1\}$;
(R4) $r(x, y)=r(n(x), n(y))$ for all $x, y\in[0, 1]$;
(R5) For all $x, y, z, t\in[0, 1]$, such that $x\le y\le z\le t$ then $r(y, z)\ge r(x, t)$.

Note that (R5) means that, for all $x, y, z\in[0, 1]$, if $x\le y\le z$ then $r(x, y)\ge r(x, z)$ and $r(y, z)\ge r(x, z)$.

REFs attempt to capture the perceived similarity between two values in [0, 1], which in fuzzy set theory usually represent membership degrees. It is usual to construct REFs from a pair of automorphisms of the unit interval, as proposed in [14], although alternative methods have also been studied [23].

Definition 3. A continuous, strictly increasing function $\varphi:[a, b]\to[a, b]$ such that $\varphi(a)=a$ and $\varphi(b)=b$ is called automorphism of the interval $[a, b]\subset\mathbb{R}$.

Proposition 1. [14] *Let φ_1, φ_2 be two automorphisms of the interval* [0, 1]. *Then*

$$r(x, y)=\varphi_1^{-1}(1-|\varphi_2(x)-\varphi_2(y)|)$$

is a REF associated with the strong negation $n(x)=\varphi_2^{-1}(1-\varphi_2(x))$.

[1] Synthetic Fingerprint Generator: http://biolab.csr.unibo.it/sfinge.html.