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Global dissipativity analysis for delayed quaternion-valued neural networks[☆]

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ABSTRACT

The problem of global dissipativity analysis for quaternion-valued neural networks (QVNNs) with time-varying delays is firstly investigated in this paper. The QVNN is studied as a single entirety without any decomposition. Several algebraic conditions ensuring the global dissipativity and globally exponential dissipativity for QVNNs are derived by employing Lyapunov theory and some analytic techniques. Furthermore, the positive invariant sets, globally attractive sets and globally exponentially attractive sets are figured out as well. Finally, the effectiveness is notarized by deducing two simulation examples.

1. Introduction

Quaternion was firstly proposed by Hamilton (1853), and it is a non-commutative division algebra. Due to the non-commutativity, the research on quaternion is much more difficult than that on real number as well as plurality, which is one of the reasons that lead to the slow development of quaternion. Fortunately, as the development of modern mathematics and the expansive applications of quaternion for future development were found in recent years, quaternion has attracted increasing attention from various areas, such as attitude control, quantum mechanics, and computer graphics (Adler, 1995; Choe & Faraway, 2004; Chou, 1992; Mukundan, 2002). Typically, quaternions are with great application prospects in three-dimensional and four-dimensional data modeling; for instance, spatial rotation in 3D geometrical affine transformations-translation can be efficiently and compactly described by quaternions (Isokawa, Matsui, & Nishimura, 2009). On the other hand, as is known to all, neural networks have been extensively applied in various areas, such as information processing, engineer optimization and automatic control and so on. In many actual applications, multidimensional data are usually encountered in actual systems, and this can be solved by adopting several real valued neurons to receive multidimensional data. However, to some extent, this treatment is unnatural because some data should not be handled separately. Two dimensional data can be operated as a whole by a complex value neuron. Analogously, scholars conceived to introduce the quaternion into neural networks to naturally describe multidimensional information, like color and three dimensional coordinates, via a quaternionic neuron. Then, the quaternion-valued neural networks (QVNNs) are obtained immediately. The states, connection weights and activation functions of QVNNs are taken as quaternion vectors or matrices. This is an extension of the research fields concerning the quaternion. It is worth pointing out that QVNNs have both the merit of quaternion and neural networks, and some exciting

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results regarding to QVNNs have been reported (Arena, Baglio, Fortuna, & Xibilia, 1995; Buchholz & Le Bihan, 2006; Isokawa, Kusakabe, Matsui, & Peper, 2003; Isokawa et al., 2009; Isokawa, Nishimura, Kamiura, & Matsui, 2007; Liu, Zhang, Lu, & Cao, 2016; Rishiyur, 2006). Arena et al. (1995) put forward the quaternionic multilayer perceptrons to predict the chaotic time series. And, it was also proved that the quaternionic multilayer perceptrons are with better prediction and smaller complexity when compared with the real multilayer perceptrons (Arena et al., 1995). The optimum separation of polarized signals was realized by QVNNs in Buchholz and Le Bihan (2006), and the simulations results showed that the performance of separation based on QVNNs is better than that based on RVNNs. Furthermore, the powerful approximate ability has also been verified in Buchholz and Le Bihan (2006). The problem of color image compression was discussed by using QVNNs with back propagation(BP) algorithm, and it was well resolved by QVNNs with BP algorithm, whereas it could not be done by RVNNs with BP algorithm (Isokawa et al., 2003). Rishiyur investigated the instantaneously trained neural networks with complex and quaternion inputs, and it was also verified that the quaternions encodings can considerably reduce the sizes of networks when compared with complex encoding in Rishiyur (2006). As a result, the amount of integrant input neurons could be dramatically reduced. The dynamics of both continuous and discrete Hopfield QVNNs have been investigated (Isokawa et al., 2007; Isokawa, Nishimura, & Matsui, 2012; Yoshida, Kuroe, & Mori, 2004). Liu et al. (2016) considered the μ-stability of QVNNs by decomposing the QVNNs into two CVNNs, and several sufficient conditions were presented in the form of linear matrix inequalities.

Dissipativity is closely related to the stability of total system, and it not only plays an important role in control theorem, but also plays a significant role in actual system, such as robotic system, electrical power system, engine system and so forth. Many scholars are full of enthusiasm on the dissipativity analysis of neural networks, and lots of meaningful results have been derived (Cao, Yuan, Ho, & Lam, 2006; Duan, Huang, & Guo, 2016; Guo, Wang, & Yan, 2013; Liao & Wang, 2003; Muralisankar, Gopalakrishnan, & Balasubramaniam, 2012; Samuel & Arcak, 2015; Song, 2011; Song & Zhao, 2005; Wu, Chen, Cao, & Lu, 2011). Liao and Wang (2003) discussed the global dissipativity of recurrent neural networks via the system parameters. Based on the lower bounds lemma and discretized Jensen inequality, some sufficient conditions for the dissipativity of discrete-time stochastic neural networks were proposed in Wu, Shi, Su, and Chu (2013). Dissipativity and robust dissipativity for neural networks with discontinuous activations have been investigated in Wu et al. (2011) and Duan et al. (2016). By employing the Lyapunov functional and inequalities techniques, Song et al. studied the dissipativity for different classes of neural networks (Song, 2011; Song & Cao, 2009; Song & Zhao, 2005). With the help of nonsmooth analysis and Filippov theory, dissipativity analysis for memristor-based neural networks was conducted as well (Guo et al., 2013; Li, Rakkiyappan, & Velmurugan, 2015). Based on the dissipativity property, Samuel and Arcak (2015) proposed a computational method for the state-space safety verification of interconnected dynamical systems. Some criteria were provided to ensure the global dissipativity for T-S fuzzy neural networks in Muralisankar et al. (2012). The dissipativity of uncertain inertial neural networks was investigated via matrix measure approach in Tu, Cao, and Hayat (2016). Ding and Shen (2016) has studied the dissipativity for the delayed fractional-order neural networks with discontinuous activations. More interesting results can be found in Arik (2004), Cao et al. (2006), Cao and Li (2017) and Chen and Huang (2004).

Time delays invariably exist in neural systems during the transmission process, and some poor performances, such as bifurcate, instability, oscillation, might be caused due to the appearance of delays. Nevertheless, some performances of dynamical systems can be improved by introducing proper delays; for instance, the synchronous ability of complex network can be enhanced by introducing proper controlling delay (Dhamala, Jirsa, & Ding, 2004). Therefore, it is necessary to investigate the dynamical behaviors of QVNNs with time delay for its theoretical significance and applications prospects. Unfortunately, the reported results focusing on the dynamics of delayed QVNNs are exiguity, and the dissipativity analysis is still challenging and unsolved as before. Motivated by the above analysis, we attempt to investigate the dissipativity for QVNNs with time varying delays.

Arrangement of the remainder part is as follows. Section 2 provides some preliminaries and model description. The dissipativity analysis for QVNNs is carried out in Section 3. In Section 4, the effectiveness of main results is verified by two numerical simulations examples. Conclusions and future works are presented in Section 5.

2. Preliminaries

In order to enhance the readability of this article, some definitions and notations are recapitulated. The quaternion is a kind of supercomplex number involving a real part and three imaginary parts i, j, k, and a quaternion x can be denoted as

$$x = x^{(r)} + x^{(i)}i + x^{(j)}j + x^{(k)}k,$$

where $x^{(r)},\ x^{(i)},\ x^{(j)}, x^{(k)} \in R$, R represents the set of real number. i, j and k satisfy the Hamilton rule, i.e.

$$i^2 = j^2 = k^2 = ijk = -1, \qquad ij = -ji = k,$$
$$jk = -kj = i, \qquad ki = -ik = j.$$

The set of quaternions denoted by $\mathbb{Q} \triangleq \{x^{(r)} + x^{(i)}i + x^{(j)}j + x^{(k)}k | x^{(r)},\ x^{(i)},\ x^{(j)}, x^{(k)} \in R\}$. $\mathbb{Q}^n$ denotes the n-dimensional quaternion space. $\bar{x} = x^{(r)} - x^{(i)}i - x^{(j)}j - x^{(k)}k$ is the conjugate of quaternion x. The modulus of $x \in \mathbb{Q}$ is defined as

$$|x| = \sqrt{x\bar{x}} = \sqrt{(x^{(r)})^2 + (x^{(i)})^2 + (x^{(j)})^2 + (x^{(k)})^2}.$$

Moreover, the norm of the quaternion vector $x = (x_1, x_2, \ldots, x_n)^T$, $x_p \in \mathbb{Q}, p = 1, 2, \ldots, n$, is defined as

$$\|x\| = \Big(\sum_{p=1}^{n} |x_p|^2\Big)^{\frac{1}{2}}.$$

$C([t_0 - \tau, t_0]; \mathbb{Q}^n)$ represents a class of continuous mapping set from $[t_0 - \tau, t_0]$ to $\mathbb{Q}^n$. For $\varphi \in C([t_0 - \tau, t_0]; \mathbb{Q}^n)$, $\|\varphi\| \triangleq \sup_{t_0-\tau\le s\le t_0} |\varphi(s)|$.

In this article, consider the delayed QVNN as follows:

$$\frac{\mathrm{d}x_p(t)}{\mathrm{d}t} = -d_p x_p(t) + \sum_{q=1}^{n} a_{pq} f_q(x_q(t)) + \sum_{q=1}^{n} b_{pq} f(x_q(t - \tau(t))) + u_p, \quad p = 1, 2, \ldots, n, \tag{1}$$

where $x_p(t) \in \mathbb{Q}$ is the state of the ith neuron at time t; d_p denotes the self-feedback coefficient satisfying $d_q > 0$; a_{pq} and $b_{pq} \in \mathbb{Q}$ are the link weights; $f_q(\cdot)$ is the activation function; $\tau(t)$ represents the time delay, and it satisfies $\dot{\tau}(t) \le \mu < 1$ and $0 \le \tau(t) \le \tau$; $u_p \in \mathbb{Q}$ is a external input. For $s \in [t_0 - \tau, t_0]$, $x_p(s) = \phi_p(s)$, $p = 1, 2, \ldots, n$, where $\phi_p(s) \in C([t_0 - \tau, t_0]; \mathbb{Q})$.

Assumption (H). There exist positive constants $l_p, p = 1, 2, \ldots, n$, for any $x_p, y_p \in \mathbb{Q}$, such that

$$|f_p(x_p) - f_p(y_p)| \le l_p |x_p - y_p|.$$