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Part 2—The firings of many neurons and their density; the neural network its connections and field of firings

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ABSTRACT

This paper is concerned with the firing of many neurons and the synthesis of these firings to develop functions and their transforms which relate chemical and electrical phenomena to the physical world. The density of such functions in the most general spaces that we encounter allows us to use linear combinations of them to approximate arbitrarily close to any phenomenon we encounter, imagine or think about. Absence of the technology needed to represent all the senses and the mathematical difficulty of making geometric representations of functions of a complex and of more general division algebra variables make it difficult to validate the mathematical outcome of this approach to neural firings. But we think that this problem will be solved in the not-too-distant future when at least the senses of smell, taste and touch would have been so mathematized that it is possible to instill these qualities in robots in some fashion.

1. Introduction

How Wise an Owl Can Be: A millipede walking through the forest developed very sore feet. In fact, he had 999 sore feet and inquired where to go to get help. The animals of the forest advised him to consult the wise owl up the big tree in the middle of the forest. After considerable effort, the millipede climbed the tree and came up to the owl and told him of his problem. The owl said, "For the next three weeks, you must walk one inch above the ground. In doing this, there would be no pressure from the ground and the surrounding air would heal your sore feet, serving both as cushion and as a gentle massage." The millipede was awed by the owl's wisdom and said, "Oh, wise owl, I am breath-taken by your wisdom and knowledge. I only have one more question to ask. How do I go about walking one inch above the ground?" The owl looked at him sternly and said, "Millipede, I have given you a conceptual solution to your problem. Don't bother me with the technical detail."

The nervous system is made up of many networks of interacting neurons. In this way, different parts of the system can "talk" to each other as well as work together to send messages to the rest of the body. All firings are described by the same type of eigenfunctions with different variables and parameters. A neuron behaves as a damped periodic oscillator of period one. Oscillation gives rise to harmonics which depend on the frequency of firing of the neurons. Our task is to synthesize the entire firings of the brain in a mathematically meaningful way (Saaty, 2000, 2015).

Thus we need a mathematical way to represent a network with flows as a feedback network system, with a hierarchy as a special case, that can capture the interactions and interdependence of the flows of electrical signals and yield a synthesized outcome. The networks in our nervous system, including both the brain, sympathetic and parasympathetic nervous systems have many neural subnets each specializing in processing a certain type of information: sensory, feelings and emotions, thoughts, actions, memories, dreams and hallucinations, and many others. How do they work collectively? Two systems of neurons may have nothing to do with each other's purpose, yet because they are interdependent and moderate each other's functional intensity, they produce overall synchrony of damped periodic oscillations.

There are several solutions of the fundamental functional equation of proportionality derived in Part 1 of this work, in the real and complex domains: continuous, differentiable, analytic, multiple valued and so on and in the quaternionic and octonionic division algebras. They can all occur at the same time in different parts of the nervous system specialized in a certain activities represented by their firings. The outcome of synthesis of each kind of firing has a synthesis that occurs in parallel with the other syntheses.

More and more science has convinced us that our world entirely consists of interconnected phenomena that are vibratory or oscillatory in nature. Therefore, our own biological brains and

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their function are also oscillatory. It follows that our physical brain, which senses everything that we feel and know, and the physical world in which we exist feel and think are intrinsically mathematical. The cosmos is influenced by four natural forces that are vibratory. From strongest to weakest, they are: strong nuclear, electromagnetic, weak nuclear, and gravity. On February 11, 2016, the LIGO Scientific Collaboration and Virgo Collaboration published a paper about the detection of gravitational waves, from a signal detected at 1.3 billion light-years from the Earth, generated by two black holes merging (see Internet), thus confirming Einstein's theory about gravity being wavelike (Abbott et al., 2016). These vibrations have abstract forms that we can only discover with mathematics. String theory in physics aims at doing this representation by using mathematically defined strings using octonions. In the 1924 Dr. Hans Berger from Austria invented a machine that proved his theory of the existence of brain waves. He called it the Electroencephalograph, or EEG for short, and used the terms alpha and beta waves.

To deal with the complexity of the vibratory interpretation, we need an abstract mathematical way to represent for our purpose, the relations between the electric phenomena of the brain. To make this presentation flow in a natural way, the next section deals with an abstract structure of the brain and its connections. Section 3 is concerned with combining functions of many neurons. It is followed in Section 4 by a discussion of density and approximation and the workings of the field generated by the firings. Section 5 concludes the exposition.

2. Matrix structure of neurons and their synaptic connections—the hypermatrix

It is a big challenge to develop the most appropriate mathematical model to describe the electrical activity of different groups of neurons. There are more neurons in the nervous system (about 86 billion) than we have seconds in a 100 year lifetime (about 3.15 billion seconds). Only a giant computer can identify and list all these neurons, their interconnection and their spike trains. To abstract these ideas mathematically, we need to think about neurons as groups performing different functions together. We also need a conceptual mathematical framework to describe the functional firing contribution of each neuron that we derived in Part 1 to the firing of other neurons with which it synapsis and these in turn to others with possible feedback and including the synapses themselves in this representation. The number of synapses may increase or decrease for each neuron over time and neural connections may also change.

The existence of a neural network that approximates any given function with a given precision was proven by Hornik, Stinchcombe, and White (1989). The utility of artificial neural network models lies in the fact that they can be used to infer a function from observations. This is particularly useful in applications where the complexity of the data or task makes the design of such a function by hand impractical. Neural nets have been successfully used to solve many complex and diverse tasks, ranging from autonomously flying aircrafts to detecting credit card fraud.

More than 10,000 different types of neurons share the same basic structure. The cell body makes proteins and regulates the neurons' metabolism. Dendrites are long, threadlike structures that bring information in from other neurons. The axon is also threadlike and carries information from the neuron to other locations. The axon has special areas on its ends that emit neurotransmitter chemicals when it receives a signal. There are many different specialized functions in the nervous system, however. This is why there are so many types of neurons. Each has a different shape, size or number of dendrites. To make classification easier, scientists have formed general classification systems. The most common system divides neurons into three types: sensory neurons, motor neurons and interneurons

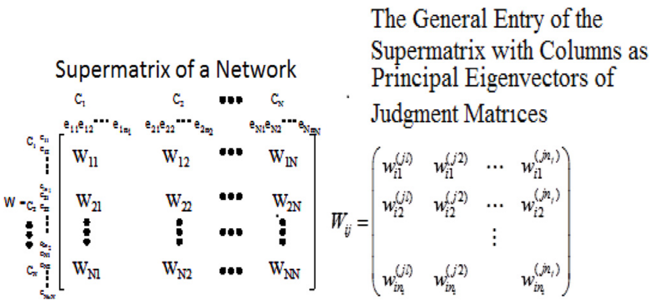

Fig. 1. Supermatrix of a network and detail of a component in it.

Neural nets synthesize the firings of many neurons, each of which does its firing or not firing individually as it is stimulated by other neurons. Sensory neurons detect incoming light, sound, odor, taste, pressure, and heat and send messages to the brain. Motor neurons transmit messages from the brain to control voluntary movement. Other parts of the nervous system regulate involuntary processes, such as the release of hormones like adrenaline, dilation of the eye in response to light, or regulation of the digestive system, which are involved in the function of the body's organs and glands.

When nerve cells communicate with each other, they do so through action potentials. For decades, the accepted idea was that they simply sum up the tiny potentials generated by the incoming pulses and emit an action potential themselves when a threshold is reached. For the first time, Moritz Helias and Markus Diesmann from the RIKEN Brain Science Institute (Japan) and Moritz Deger and Stefan Rotter from the Bernstein Center Freiburg (Germany) (2010) now explain what exactly happens right before a nerve cell emits a pulse. Not only does this theory explain why nerve cells process information much faster than previously thought. It also became clear that neurons do more than just add up pulses: In the decisive moments, they actually **multiply**. They write that the availability of this mathematical operation finally explains how the brain is able to execute complex computations. These insights in the basic processes of the brain will in turn inspire more powerful processor architectures in the future (see Internet).

In decision making we use eigenvector priorities for the alternatives of a decision (Saaty, 2010). We then take linear combinations of these eigenvectors by weighting and adding. The multiplication constants in the linear combination are the priorities of the criteria used to evaluate the alternatives. So it is for the firing of neurons with eigenfunctions but the multipliers are the eigenfunctions of other neurons rather than constants. All firings are described by the same form of eigenfunctions with different variables and parameters. A neuron behaves as a damped periodic oscillator of period one. Oscillation gives rise to harmonics which depend on the frequency of firing of the neurons. Our task is to synthesize the entire firings of the brain in a mathematically meaningful way (Saaty, 2000, 2015).

We propose representing the nervous system with a large matrix we call a hypermatrix whose entries are matrices that we call supermatrices which we have been using extensively in making complex decisions. The entries of a supermatrix are themselves smaller matrices whose entries are column eigenvectors (eigenfunctions in the case of neural firings).

A supermatrix along with an example of one of its general entry matrices is shown in Fig. 1.

Much is known about functions of matrices. The theory that is known in using functions of matrices is the Perron Frobenius theorem for example which holds pointwise. Not as much is known about matrices whose entries are functions.

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