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Mean-square exponential input-to-state stability of delayed Cohen–Grossberg neural networks with Markovian switching based on vector Lyapunov functions

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ABSTRACT

This paper studies the mean-square exponential input-to-state stability of delayed Cohen–Grossberg neural networks with Markovian switching. By using the vector Lyapunov function and property of M-matrix, two generalized Halanay inequalities are established. By means of the generalized Halanay inequalities, sufficient conditions are also obtained, which can ensure the exponential input-to-state stability of delayed Cohen–Grossberg neural networks with Markovian switching. Two numerical examples are given to illustrate the efficiency of the derived results.

1. Introduction

In 1983, Cohen and Grossberg presented a neural network (Cohen & Grossberg, 1983), which is now called Cohen–Grossberg neural network (CGNN). As we know, Cohen–Grossberg neural networks include the famous Hopfield neural networks, cellular neural networks as its special cases. This model has been paid much considerable attention due to its wide applications in various areas such as pattern classification, associative memory, parallel computation, optimization, system identification and control, moving object speed detection and so on. Accordingly, a great number of results have been published concerning CGNNs (Cao & Liang, 2004; Song & Cao, 2006; Yuan, Cao, & Li, 2006; Zhou, Teng, & Xu, 2015; Zhu, Cao, & Rakkiyappan, 2015; Zhu & Li, 2012) and the references cited therein.

Time delays are unavoidable in hardware implementation owing to the finite switching speed of amplifiers. It may often lead to the oscillation, divergence, and even instability during the application of neural networks. In the past few years, delayed neural networks have been extensively studied by researchers and there have appeared a large number of results in the literature, for instance (Cao & Liang, 2004; Long & Xu, 2013; Xu, Luo, Zhong, & Zhu, 2014; Zhou et al., 2015; Zhu et al., 2015) and references therein. In addition, since Markovian jump linear systems were firstly introduced in early 1960s, various systems driven by continuous time Markovian chains have been widely employed to practical systems where they may experience abrupt changes in system structure and parameters. In such a case, neural networks can be represented by a switching model which can be regarded as a set of parametric configurations switching from one to another according to a given Markovian chain (see Huang, Ho, & Qu, 2007; Mao & Yuan, 2006; Shen & Wang, 2009; Zhu & Cao, 2012; Zhu & Cao, 2010; Zhu & Cao, 2011). Taking the time delays and Markovian switching into account, it is actually valuable to investigate the stability of delayed Cohen–Grossberg neural networks (DCGNNs) with Markovian switching.

It is well-known that the stability of neural network is not only the most basic and important problem but also the foundation of neural network's applications. Recently, there have been a lot of literature on the stability analysis of neural networks reported in the literature (see Liu, Shen, & Jiang, 2011; Liu, 2016; Shen & Wang, 2008; Wang, Shen, & Ding, 2015; Xiao, Zeng, & Wu, 2014; Zhang & Shen, 2015; Zhu, Zhong, & Shen, 2014). On the

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other hand, the input-to-state stability (ISS) is one of the useful properties for nonlinear systems (see Zhou et al., 2015; Zhu et al., 2015). The ISS concept means that no matter what the initial state is, if the inputs are uniformly small, then the state of the neural networks must eventually be small. It offers an effective way to handle the stabilization of neural network applications in the presence of various uncertainties. Recently, some results on the ISS properties are obtained for neural networks. For example, Sanchez and Perez (1999) investigated the ISS properties and gave some matrix norm conditions on ISS of recurrent neural networks (RNNs) firstly. Ahn utilized Lyapunov function method to discuss robust stability problem for a class of RNNs, and also some LMI sufficient conditions have been proposed to guarantee the ISS in Ahn (2011). It is Zhu and Cao who firstly investigated the exponential ISS for stochastic neural networks in Zhu and Cao (2014) and Zhu et al. (2015). Recently, we have also noticed that some scholars begin to discuss the exponential stability of neural networks by means of the vector Lyapunov function methods, see e.g. Long and Xu (2013), Shen and Wang (2009) and Zhou et al. (2015). For example, Shen and Wang (2009) have considered the exponential stability of DRNNs with Markovian switching by a generalized vector Halanay inequality. By using the Razumikhin technique, two sufficient criteria on mean square exponential ISS of stochastic delayed exponential ISS of are derived in Zhou et al. (2015). It is well-known that the existing criteria cannot handle the exponential ISS of DCGNNs with Markovian switching, since the simultaneous presence of external input item and switching mechanism. To the best of our knowledge, the exponential ISS of DCGNNs with Markovian switching has scarcely been investigated. As such, this issue constitutes the first motivation of this paper.

Moreover, for most of the existing stability criteria for neural networks, described by scalar Lyapunov function or vector Lyapunov function, the time-delayed item and the non-delay item are separated in corresponding $\mathscr{L}$-operator differential inequality. It is well-known that the cross items will inevitably arise when using the Lyapunov function to study the stability of delayed systems. While most scholars choose to use the elemental inequality to deal with the cross items. As a result, these proposed methods seem to be more conservative. Naturally, an interesting question is generated whether the cross items could arise in the vector $\mathscr{L}$-operator differential inequality? Solving this problem is the second motivation of this paper.

Summarizing the above statements, the focus of this paper is to discuss the exponential ISS for DCGNNs with Markovian switching, and obtain the criteria described by vector $\mathscr{L}$-operator differential inequality with cross items. The main contributions of this paper lie in two aspects: (1) By using the vector Lyapunov functions and stochastic analysis technique, two generalized stochastic vector Halanay inequalities are established; (2) Based on the novel Halanay inequalities, sufficient algebraic criteria with less conservative are obtained to ensure the ISS in mean square sense.

The remainder of this paper is organized as follows. Section 2 introduces the model of DCGNNs with Markovian switching and gives some necessary notations. Section 3 presents the main results. Two numerical examples are given to show the effectiveness of the main results in Section 4. Finally, concluding remarks are made in Section 5.

2. Models and preliminaries

Throughout this paper, unless otherwise specified, we let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathscr{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., it is right continuous and $\mathcal{F}$ contains all P-null sets). Let $\tau > 0$ and $C = C([-\tau, 0], R^n)$ denote the family of continuous functions φ from $[-\tau, 0]$ to R^n with the norm $\|\varphi\|_\tau = \sup_{-\tau\leq\theta\leq 0}|\varphi(\theta)|$, where $|\cdot|$ is the Euclidean norm in R^n. Denote by $\mathcal{L}^2_{\mathcal{F}_t}$ the family of all $C([-\tau, 0]; R^n)$-valued, $\mathcal{F}_t$-adapted stochastic variables $\phi = \{\phi(s), -\tau \leq s \leq 0\}$ such that $\int_{-\tau}^0 E|\phi(s)|^2 ds < \infty$, where E stands for the correspondent expectation operator with respect to the given probability measurable $\mathscr{P}$. The set of all essentially bounded functions $u : R_+ \to R^n$, endowed with essential supremum norm $\|u\|_\infty = \sup\{|u(t)|, t \geq 0\}$, is denoted by L^n_∞. A function $\varphi : R_+ \to R_+$ is said to be class of $\mathcal{K}$ if it is continuous and strictly increasing and satisfies $\varphi(0) = 0$; it is of class $\mathcal{K}_\infty$ if in addition $\varphi(s) \to \infty$ as $s \to \infty$. Let G is a vector or matrix. By $G \geq 0$ we mean that each element of G is non-negative. By $G \gg 0$ we mean that all elements of G are positive. And if $G = (g_{ij})_{n\times n} \in R^{n\times n}$, we denote $|G| = (|g_{ij}|)_{n\times n}$, $\bar{G} = (\bar{g}_{ij})_{n\times n}$ with $\bar{g}_{ij} = g_{ij}(i \neq j), \bar{g}_{ii} = g^+_{ii} = \max\{g_{ii}, 0\}$, $i = 1, \ldots, n$. Moreover, we also adopt here the traditional notation by letting

$$Z^{n\times n} = \{A = \{a_{ij}\}_{n\times n} : a_{ij} \leq 0, i \neq j\}.$$

Let $\{r(t)(t \geq 0)\}$ be a right-continuous Markovian chain on the probability space taking values in a finite state space $\mathcal{M} = \{1, 2, \ldots, N\}$ with generator $\Gamma = (\gamma_{ij})_{N\times N}$ given by

$$P\{r(t+\Delta) = j|r(t) = i\} = \begin{cases} \gamma_{ij}\Delta + o(\Delta) & i \neq j \\ 1 + \gamma_{ii}\Delta + o(\Delta) & i = j, \end{cases}$$

where $\Delta > 0$. Here $\gamma_{ij} \geq 0$ is the transition rate from i to j if $i \neq j$ while $\gamma_{ii} = -\Sigma_{i\neq j}\gamma_{ij}$.

In this paper, we consider a DCGNN with Markovian switching

$$dx_k(t) = h_k(x_k(t))\Big[-d_k(r(t), x_k(t)) + \sum_{l=1}^n a_{kl}(r(t))f_l(x_l(t)) + \sum_{l=1}^n b_{kl}(r(t))f_l(x_l(t-\tau_l(t))) + u_k(t)\Big]dt, \quad k = 1, 2, \ldots, n, \tag{1}$$

for $t \geq 0$ with initial value $\xi \in C$, $r_0 \in \mathcal{M}$, where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ is the state of the neuron at time t, $H(x(t)) = \mathrm{diag}(h_1(x_1(t)), h_2(x_2(t)), \ldots, h_n(x_n(t)))$ represents the amplification function of the neuron at time t, $D(i, x(t)) = \mathrm{diag}(d_1(i, x_1(t)), d_2(i, x_2(t)), \ldots, d_n(i, x_n(t)))^T$ is the appropriately behaved function dependent on t and on the state processes $x(t)$, while $A(i) = (a_{kl}(i))_{n\times n}$ and $B(i) = (b_{kl}(i))_{n\times n}$ describe the connection weight matrices associated without delays and with delays, respectively. $\tau_l(\cdot)$ $l = 1, \ldots, n$ denote the time-varying delay, that satisfies $0 \leq \tau_l(t) \leq \tau$, where τ is the maximal delay. $U(t) = (u_1(t), u_2(t), \ldots, u_n(t))^T$ is the external input function at time t, $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t))^T)$ and $f(x(t-\tau(t))) = (f_1(x_1(t-\tau_1(t))), f_2(x_2(t-\tau_2(t))), \ldots, f_n(x_n(t-\tau_n(t))))^T$ are vector-valued activation functions.

In order to prove our main results, we make the following assumptions on the amplification functions, behaved functions, and activation functions.

Assumption 1. There exist positive constants $\underline{h}_k, \overline{h}_k$ such that

$$0 < \underline{h}_k \leq h_k(x) \leq \overline{h}_k, \quad x \in R,\ k = 1, 2, \ldots, n.$$

Assumption 2. For $i \in \mathcal{M}$, there exist positive constants $\delta_k(i)$ such that

$$xd_k(i, x) \geq \delta_k(i)x^2, \quad x \in R,\ k = 1, 2, \ldots, n.$$

Assumption 3. There exist positive constants M_k such that

$$0 \leq \frac{f_k(x) - f_k(y)}{x - y} \leq M_k, \quad x, y \in R,\ k = 1, 2, \ldots, n.$$