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# Multistability of Complex-Valued Neural Networks with Discontinuous Activation Functions

Jinling Liang*, Weiqiang Gong and Tingwen Huang

**Abstract**

In this paper, based on the geometrical properties of the discontinuous activation functions and the Brouwer's fixed point theory, the multistability issue is tackled for the complex-valued neural networks with discontinuous activation functions and time-varying delays. To address the network with discontinuous functions, Filippov solution of the system is defined. Through rigorous analysis, several sufficient criteria are obtained to assure the existence of $25^n$ equilibrium points. Among them, $9^n$ points are locally stable and $16^n - 9^n$ equilibrium points are unstable. Furthermore, to enlarge the attraction basins of the $9^n$ equilibrium points, some mild conditions are imposed. Finally, one numerical example is provided to illustrate the effectiveness of the obtained results.

**Index Terms**

Complex-valued neural networks, multistability, discontinuous function, attraction basin.

## I. Introduction

The dynamic behaviors of the complex-valued networks have recently received much attention due to their useful applications, such as associative memory, filtering, image processing, computer vision, optoelectronics, speech synthesis, and so on, see [1]–[9] and the references therein for more information. Different from the real-valued networks, all the states and connection weights of the complex-valued networks are complex-valued, which makes them better than the real-valued ones when processing the complex signals. Generally speaking, the complex-valued networks have much more complicated properties than the real-valued ones in many aspects. Take the XOR problem and the detection of symmetry problem for example, both of them cannot be solved with a real-valued signal neuron, however, which could be easily solved by a single complex-valued neuron with orthogonal decision boundaries [10], [11]. Hence, it is important to explore the dynamics of the complex-valued networks, especially the stability issue [12]. Up till now, lots of complex-valued models have been developed and extensively investigated, see [13]–[16] and the references cited therein. In [17], the complete stability, global attractivity, and boundedness have been discussed for the discrete-time neural networks with complex-valued linear threshold neurons. A new stability condition has been derived in [18] for the complex-valued neural network by the energy function approach, and the obtained criterion has not only permitted a little relaxation on the Hermitian assumption of the connection matrix but also generalized some existing results.

It is well known that the number of equilibrium points depends on the practical applications of neural networks. Some applications, such as the optimization, require the network to have only one equilibrium point which is monostable. However, the multiple/stable equilibrium points of neural networks are demanded in some other

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