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Complete Stability of Delayed Recurrent Neural Networks with Gaussian Activation Functions

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Abstract

This paper addresses the complete stability of delayed recurrent neural networks with Gaussian activation functions. By means of the geometrical properties of Gaussian function and algebraic properties of nonsingular M-matrix, some sufficient conditions are obtained to ensure that for an n-neuron neural network, there are exactly 3^k equilibrium points with $0 \leq k \leq n$, among which 2^k and $3^k - 2^k$ equilibrium points are locally exponentially stable and unstable, respectively. Moreover, it concludes that all the states converge to one of equilibrium points; i.e., the neural networks are completely stable. The derived conditions herein can be easily tested. Finally, a numerical example is given to illustrate the theoretical results.

Keywords: Recurrent neural networks, complete stability, time-varying delays, Gaussian functions

1. Introduction

In recent years, the dynamic analysis of neural networks has received considerable attention due to the fact that neural networks have been successfully applied to solve various practical engineering problems. Some problems allow that neural networks just have a unique equilibrium point and all the trajectories of neural network converge to the point. Yet many problems, such as pattern recognition, classification, associative memories and so on, require that neural networks possess multiple locally stable equilibrium points. Hence, other than mono-stability, which means that neural networks have just a unique and globally stable equilibrium point, multistability, which means the coexistence of multiple equilibrium points and their corresponding local stability, is also an important research topic.

It is generally agreed that the type of activation functions plays a crucial role in the multistability analysis of neural networks. Different types of activation functions might cause different dynamic behaviors of neural networks. There are some types of widely used activation functions, such as threshold function, saturation function, sigmoid function, the Gaussian function, the Mexican-hat function and so on. The mono-stability analysis of neural networks with these types of activation functions has been investigated extensively, e.g. Cohen & Grossberg (1983); Arik (2000); Zhang & Han (2009); Zhang et al. (2014); Wang & Wu (2014); Wen et al. (2015); Zhu et al. (2015); Zhang et al. (2015); Feng & Zheng (2015); Zhu & Cao (2010, 2011, 2012). However, not all types of activation functions are considered in the multistability analysis of neural networks. Specifically, the multistability of recurrent neural networks with saturation standard activation functions is addressed in Zeng et al. (2004, 2005). The results in Zeng et al. (2004, 2005) are extended to general piecewise linear nondecreasing activation functions Wang et al. (2010); Nie & Cao (2011); Zeng & Zheng (2012, 2013). Moreover, the multistability of neural networks with the piecewise linear Mexican-hat-type activation functions Wang & Chen (2012b) and discontinuous piecewise-linear activation functions Bao & Zeng (2012); Huang et al. (2012); Nie & Zheng (2015) are discussed. In addition, the existence of 3^n stationary solutions for the n-dimensional delayed neural networks with a general class of sigmoidal activation functions is analyzed in Cheng et al. Cheng et al. (2007, 2006) and it is shown that 2^n of equilibria are stable. The results in Cheng et al. (2015) are extended to show the existence of 3^k equilibria with $k \leq n$, among which 2^k equilibria being attractive in an n-neuron network with the same activation functions. The multistability of neural networks with a general class nonlinear and nondecreasing activation functions is studied in Wang & Chen (2014). More results can be found in Shayer & Campbell (2000); Kaslik & Sivasundaram (2011); Zeng & Zheng (2012); Di Marco et al. (2014) and references therein.

In view that the activation functions considered by the existing works as mentioned above are restricted to be either nondecreasing or piecewise linear, the multistability of recurrent neural networks with nonmonotonic activation functions and mixed time delays is analyzed paper Liu et al. (2016b), showing that such an n-neuron neural network can have 3^n equilibrium points, among them 2^n ones are locally exponentially stable. And in Liu et al. (2016a), the multistability of a general class of recurrent neural networks with non-monotonic activation functions and time-varying delays is investigated. However, there

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