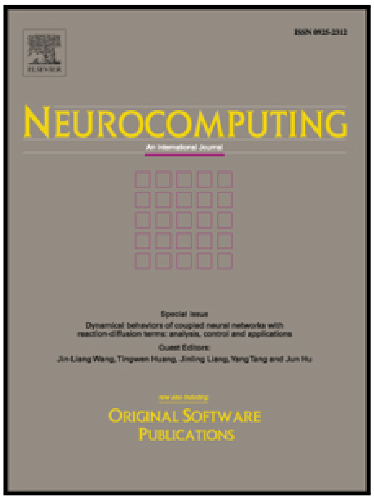

Fast Centralized Integer Resource Allocation Algorithm and Its Distributed Extension over Digraphs

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***Abstract*—This paper studies the resource allocation problem with convex objective functions, subject to individual resource constraints, equality constraints, and integer constraints. The goal is to minimize the total cost when allocating the total resource D to n agents. We propose a novel min-heap and optimization relaxation based centralized algorithm and prove that it has a computational complexity of $\mathcal{O}(n \log n + n \log D)$ when the resource constraints of individual agents are $[0, D]$, which outperforms the best known multi-phase algorithm with $\mathcal{O}(n \log n \log D)$. By extending the centralized algorithm, we present a consensus based distributed optimization algorithm to solve the same problem. It is shown that the proposed distributed algorithm converges to a global minimizer provided that the digraph (representing the interaction topology of the agents) is strongly connected. All the updates used in the distributed algorithm rely only on local knowledge.**

***Index Terms*—Min-heap, Resource allocation, Distributed algorithm, Consensus, Strongly connected digraph.**

I. Introduction

The resource allocation problem (RAP) deals with how to allocate available resources to a number of users, called agents. Many optimization problems in the financial markets, smart grids, wireless sensor networks, military ad hoc networks, and cloud systems, can be modelled as a RAP. Examples include economic dispatch problems [1], power regulation [2] and take or pay fuel supply problems [3]. Moveover, in many practical examples (e.g., joint replenishment problem [4], software-testing resources allocation [5], and many others in [6]–[8]), the resources allocated to agents can only be integer numbers.

To formulate the integer resource allocation problem(iRAP), consider a network of n agents. For each agent $i \in \{1, \cdots, n\}$, we associate a variable $x_i \in \mathbb{N}$ and a corresponding cost function $F_i(x_i)$. The iRAP is the optimization problem aiming to find an optimal integer x_i to $\underset{x_1,\ldots,x_n}{\text{minimize}} \quad \sum_{i=1}^{n} F_i(x_i)$ under a collective equality constraint $\sum_{i=1}^{n} x_i = D$ and individual inequality (state) constraints $\underline{x}_i \leq x_i \leq \overline{x}_i$, where $\underline{x}_i, \overline{x}_i$ and D are given integers. The physical description of D is that there exists a fixed homogeneous pool with D resource units. The lower and upper bounds $\underline{x}_i$ and $\overline{x}_i$ indicate the capability of each agent.

Although many centralized optimization algorithms exist for this problem, it is to reduce the computational complexity that motivates us to revisit the problem. Besides, due to significant communications overhead required for collecting information from all the agents in large-scale networks and lacking of robustness and privacy (by requiring individual agents to provide information) [1], it is desired to design distributed algorithms for solving the iRAP. Thus in this paper, we first design a fast centralized algorithm for solving the iRAP and analyze its computational complexity, and next design a distributed algorithm for the iRAP based on only locally available information.

Since the iRAP with integer constraints can be exactly solved by a simple greedy algorithm with a computational complexity of $\mathcal{O}(D \log n)$ [9], the study of computational complexity becomes one of the main issues for the iRAP. In real applications, D is always far greater than n. Thus, the problem to reduce the computational complexity $\mathcal{O}(D \log n)$ is to reduce the complexity factor caused by D. [10] proposes a Lagrange multiplier based algorithm with a computational complexity of $\mathcal{O}(n^2(\log D)^2)$, which is better than $\mathcal{O}(D \log n)$ when $D \gg n^2$. In addition, a polynomial-time algorithm [11] is proposed and runs in $\mathcal{O}(n(\log D)^2)$, which is far better than the algorithm in [10] when $D \gg n \gg 1$. More recently, [12] proposes a multi-phase algorithm (a modified polynomial-time algorithm) that reduces the computational complexity by requiring $\mathcal{O}(n \log n \log D)$.

The first key theoretical contribution of our work is the proposition of a novel min-heap and optimization relaxation (iRAP relaxation) based centralized algorithm that runs in $\mathcal{O}(n \log n + n \log D)$, which substantially reduces the computational complexity in comparison with the existing algorithms in the literature. We first notice the fact that the relaxation for the iRAP (iRAP without integer constraints) converges exponentially, which means an approximate relaxation solution can be obtained when it runs in $\mathcal{O}(nlognD)$ with the state constraints $0 \leq x_i \leq D$. Besides, we explore the relation between the relaxation solution and the optimal solution, based on which we design a method to adjust the relaxation solution to an optimal solution. We show that the adjusting procedure runs in n iterations. In addition, during the adjusting procedure,

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