Neurocomputing 000 (2017) 1–6

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Probabilistic hypergraph matching based on affinity tensor updating

Xu Yang[a], Zhi-Yong Liu[a,b,c,*], Hong Qiao[a,b,c], Jian-Hua Su[a]

[a] State Key Laboratory of Management and Control for Complex Systems, Institute of Automation, Chinese Academy of Sciences, Beijing 100190, China
[b] Center for Excellence in Brain Science and Intelligence Technology, Chinese Academy of Sciences, Shanghai 200031, China
[c] University of Chinese Academy of Sciences, Beijing 100049, China

A R T I C L E I N F O

Article history:
Received 30 September 2016
Revised 22 December 2016
Accepted 23 December 2016
Available online xxx

Keywords:
Hypergraph matching
Probabilistic graph matching
Tensor decomposition
Structural pattern recognition

A B S T R A C T

Graph matching is a fundamental problem in artificial intelligence and structural data processing. Hypergraph matching has recently become popular in the graph matching community. Existing hypergraph matching algorithms usually resort to the continuous methods, while the combinatorial nature of hypergraph matching is not well considered. Therefore in this paper, we propose a novel hypergraph matching algorithm by introducing the affinity tensor updating based graduated projection. Specifically, the hypergraph matching problem is first formulated as a combinatorial optimization problem in a high order polynomial form. Then this NP-hard problem is relaxed and interpreted in a probabilistic manner, which is approximately solved by iterative techniques. The updating of the affinity tensor is performed in each iteration, besides the updating of probabilistic assignment vector. Experimental results on both synthetic and real-world datasets witness the effectiveness of the proposed method.

1. Introduction

Graph matching, aiming to find an optimal set of vertex assignments between graphs, is a fundamental problem in artificial intelligence and structural data processing, such as the biomedical & biological applications, local feature feature correspondence, CAD image analysis, etc.

Most graph matching problems are NP-complete problems,[1] and there have been various types of algorithms to approximately solve them. Please refer to [2] for a comprehensive survey on the history and categorization of these algorithms. Generally, the main research stream has been focused on the second-order graph matching problem, which explores the pairwise structural information of graphs. Some representative pairwise graph matching algorithms include the spectral methods [3–5], the deterministic methods [6–8], the convex optimization methods [9], etc.

In recent years, promoted by the fast increasing computational ability and storage capacity of computers, hypergraph matching which explores high-order relations between vertices beyond the pairwise cues, has attracted the attentions of many leading researchers in the graph matching community. The existing hypergraph matching algorithms usually resort to the continuous methods, which first relax the original combinatorial optimization problem to a continuous optimization problem, and then project the continuous solution back to the discrete domain.

However, the combinatorial nature of hypergraph matching is usually not considered in the optimization processes of these algorithms. On the other hand it has been observed that incorporating discrete constraints in the optimization process may greatly improve the matching performance in pairwise graph matching algorithms.

Therefore in this paper, we propose a novel hypergraph matching algorithm by introducing the affinity tensor updating based graduated projection. Specifically, the hypergraph matching problem is first formulated as a combinatorial optimization problem in a high order polynomial form. Then this NP-complete problem is relaxed and interpreted in a probabilistic manner, which is approximately solved by iterative techniques. The updating of the affinity tensor, which encodes high-order similarity between hypergraphs, is performed in each iteration, besides the updating of probabilistic assignment vector. Consequently, the final probabilistic assignment vector obtained is usually very close to the discrete domain. Experimental results on both synthetic and real-world datasets witness the effectiveness of the proposed method.

The remaining manuscript is organized as follows: After some discussions on the related works in Section 2, the proposed hypergraph matching algorithm is introduced in Section 3, which is followed by the experimental result analysis in Section 4. Finally Section 5 concludes the paper.

* Corresponding author.
E-mail addresses: xu.yang@ia.ac.cn (X. Yang), zhiyong.liu@ia.ac.cn (Z.-Y. Liu), hong.qiao@ia.ac.cn (H. Qiao), jianhua.su@ia.ac.cn (J.-H. Su).

[1] One exception is the graph isomorphism problem, which has the potential to be solved in polynomial time [1].

http://dx.doi.org/10.1016/j.neucom.2016.12.096
0925-2312/

2. Related works

In this section, a brief survey on the hypergraph matching algorithms is first presented, and then the graduated projection strategies in recent graph matching algorithms, especially the pairwise graph matching algorithms, are discussed.

2.1. Hypergraph matching algorithms

The first hypergraph matching algorithm was proposed by Zass and Shashua [10] in a probabilistic framework, which assumes that the unfolded affinity tensor is equal to the Kronecker product of assignment matrix. Duchenne et al. [11] extended the famous spectral method [3] to the tensor situation by computing the rank-one approximation of the affinity tensor. Simultaneously, Chertok and Keller [12] also proposed a high order spectral algorithm based on the unfolded affinity tensor. The integer projected fixed point (IPFP) [13] algorithm proposed by Leordeanu et al. was generalized to the high order situation by the same authors in a semi-supervised learning framework, which is named by the IPFP-3D [14]. Similarly, the reweighted random walk matching (RRWM) algorithm was also generalized to hypergraph matching problem by the same research team, which is named by the RRWHM [15]. Focusing on the high complexity caused by the affinity tensor, Park et al. [16] proposed a novel method to reduce the redundancy in the affinity tensor. Celiktutan et al. [17] proposed a dynamic programming based exact optimization algorithm for the relaxed hypergraph matching model, which is used in action recognition. Nguyen et al. [18] reformulated the original polynomial objective function associated with the hypergraph matching model by a higher order convex multi-linear form, and solved it based on a tensor block coordinate scheme. Yan et al. [19] devised a discrete hypergraph matching scheme by introducing a discretization step in each iteration with theoretically guaranteed convergence property.

2.2. Graduated projection strategies in graph matching

In recent graph matching algorithms, it has been observed that introducing the graduated projection of the continuous solution to the discrete domain could significantly improve the matching performance. One first algorithm considering the graduated projection may be the well-known graduated assignment, which is still considered to be state-of-the-art [4,20]. Besides, by taking advantage of the convex relaxation and concave relaxation of the objective function, the path following algorithm [7,21], has attracted the attentions of many researchers, which gradually push the continuous solution to be a discrete one. Later, the path following algorithm was extended and improved by [8,22–24] with a similar idea. The graduated projection is also introduced to the important group of spectral graph matching methods which explore the spectral characteristics of the graphs. Specifically, by generalizing the conditional gradient ascent [25,26] method, a discrete (integer) projection scheme is introduced in [13], which is named by IPFP as mentioned above. By introducing a reweighted step, the RRWM is also related to the graduated projection, which, to some extent, can be treated as a combination the classic spectral method [3] and the graduated assignment method [6]. The recently proposed pairwise graph matching algorithm [4] also achieves a type of graduated projection by iteratively refining the affinity matrix and the softened assignment vector.

Representative hypergraph matching algorithms considering the graduated projection include the above mentioned IPFP-3D and RRWHM.

3. Probability hypergraph matching algorithm

3.1. Problem formulation

A hypergraph $\mathcal{G} = \{V, L, E, W\}$ of size M is defined by a vertex set $V = \{1, \ldots, M\}$, a vertex label set $L = \{l_1, \ldots, l_M\}$, a finite hyper-edge set $E = \{e_1, e_2, \ldots\}$, and a finite hyper-edge weight set $W = \{w_{e_1}, w_{e_2}, \ldots\}$. A vertex label $l_i \in \mathbb{R}^{d_L}$ refers to a descriptor vector for the vertex i. For instance, if graphs are constructed from the SIFT feature points [27] in image processing problems, the $d_L = 128$ dimensional appearance descriptor can be used as the vertex label. Different from the usual graph edge $e = ij$ which connects two vertices i and j, a hyper-edge $e_1 = ij\ldots$ could connect any number of vertices. If all the hyper-edges in a hypergraph connect an equal number of vertices, or say they are the same size k, the hypergraph is named by the k-uniform hypergraph. For instance, a 2-uniform hypergraph is a usual graph. It is straightforward that a hypergraph matching problem can be transformed into a combination of a series of hypergraph matching subproblems, which subproblem matches two k-uniform hypergraphs. Since the proposed algorithm are applicable to any k-uniform hypergraph matching problem, for representation and reading convenience this paper only focuses on the 3-uniform hypergraph, without considering the pairwise and higher-order relations. In this case, a hyper-edge $e_1 = ijk$ is a triplet, and a hyper-edge weight $w_{ijk} \in \mathbb{R}^{d_w}$ is vector describing the structural property of the triplet ijk.

Given two hypergraphs $\mathcal{G}$ of size M and $\mathcal{H}$ of size N, a matching between them is to find an optimal set of assignments between the two vertex sets $V^{\mathcal{G}}$ and $V^{\mathcal{H}}$. Mathematically, these assignments can be represented by an assignment matrix $\mathbf{X} \in \{0, 1\}^{M \times N}$. Specifically, $\mathbf{X}_{i,a} = 1$ denotes that there exist an assignment $\{i, a\}$ between vertex i in $\mathcal{G}$ and vertex a in $\mathcal{H}$, and $\mathbf{X}_{i,a} = 0$ means that no assignments exist between the two vertices. When further considering the one-to-one matching constraint, which is a common assumption in graph matching problems, $\mathbf{X}$ can be defined by

$$\mathbf{X} \in \mathcal{D} = \left\{ \mathbf{X} | \sum_i \mathbf{X}_{i,a} \le 1, \ \sum_a \mathbf{X}_{i,a} \le 1, \mathbf{X}_{i,a} \in \{0, 1\}. \right\} \quad (1)$$

The row-wise replica of $\mathbf{X}$ is denoted by $\mathbf{x}$, i.e. $\mathbf{x}_{(i-1)N+a} = \mathbf{X}_{i,a}$.

The similarity between $\mathcal{G}$ and $\mathcal{H}$ can be encoded by a non-negative third order tensor $\mathbf{A} \in [0, +\infty)^{MN \times MN \times MN}$ named by the affinity tensor, which is a generalization of the affinity matrix in pairwise graph matching algorithms [3,4]. Mathematically, $\mathbf{A}$ can be defined as follows:

$$\begin{aligned} \mathbf{A}_{i',j',k'} &= \mathbf{A}_{(i-1)N+a, \ (j-1)N+b, \ (k-1)N+c} \\ &= \begin{cases} (1-\alpha)\mathcal{A}(l_i, l_a), & \text{if } i = j = k, a = b = c, \\ \alpha \mathcal{A}(w_{ijk}, w_{abc}), & \text{if } ijk \text{ in } \mathcal{G} \text{ and } abc \text{ in } \mathcal{H} \text{ both exist}, \\ 0, & \text{otherwise}. \end{cases} \end{aligned} \quad (2)$$

where the diagonal entry $\mathcal{A}(l_i, l_a)$ is an affinity measure of two labels l_i in $\mathcal{G}$ and l_a in $\mathcal{H}$, and the non-diagonal entry $\mathcal{A}(w_{ijk}, w_{abc})$ is an affinity measure of two weights w_{ijk} in $\mathcal{G}$ and w_{abc} in $\mathcal{H}$. The weight parameter α is used to balance the two types of affinity measures.

Typically, the matching between $\mathcal{G}$ and $\mathcal{H}$ can be formulated by the maximization of a cubical term as follows:

$$\begin{aligned} \mathbf{x} &= \arg\max_{\mathbf{x}} \sum \mathbf{A}_{(i-1)N+a,(j-1)N+b,(k-1)N+c} \mathbf{x}_{(i-1)N+a} \mathbf{x}_{(j-1)N+b} \mathbf{x}_{(k-1)N+c} \\ &= \mathbf{A} \otimes_I \mathbf{x} \otimes_J \mathbf{x} \otimes_K \mathbf{x}. \end{aligned} \quad (3)$$

where I, J, and K denote the three dimensions of $\mathbf{A}$, and the notation $\otimes_I$ denotes the mode-I product of a tensor and a vector