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# Modeling of nonlinear systems using the self-organizing fuzzy neural network with adaptive gradient algorithm

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ABSTRACT

In this paper, a self-organizing fuzzy neural network with adaptive gradient algorithm (SOFNN-AGA) is proposed for nonlinear systems modeling. First, a potentiality of fuzzy rules (PFR) method is introduced by using the output of normalized layer and the error reduction ratio (ERR) in the training process. And a structure learning approach is developed to determine the network size based on PFR. Second, a novel adaptive gradient algorithm (AGA) with adaptive learning rate is designed to adjust the parameters of SOFNN-AGA. Moreover, a theoretical analysis on the convergence of SOFNN-AGA is given to show the efficiency in both fixed structure and self-organizing structure cases. Finally, to demonstrate the merits of SOFNN-AGA, simulation and experimental results of several benchmark problems and a real world application are examined for nonlinear systems modeling with comparisons against other existing methods. Some promising results are reported in this study, indicating that the proposed SOFNN-AGA performs better favorably in terms of both convergence speed and modeling accuracy.



## 1. Introduction

IT is well known that most real-world dynamic systems are nonlinear. Thus, modeling of nonlinear systems has attracted a lot of attention in the past decades (see [1–3] and the references therein). Modeling of nonlinear systems is to build a mathematical regressive model with measurement data, and many methods have been proposed, such as fuzzy systems, neural networks, support vector regression, and nonlinear autoregressive moving average with exogenous input methods [4–6]. Among these methods, fuzzy neural network (FNN) is a promising one, which combines the learning ability of neural networks and the interpretability of rule-based fuzzy systems [7–8]. However, in the practical applications, there are two main problems for designing FNNs: (1) how to determinate the network size, and (2) how to adjust the parameters.

For determining the size of FNN, many self-organizing FNNs have been proposed [9–10]. For example, Wang et al. proposed a fast and accurate online self-organizing scheme for parsimonious FNN (FAOS-PFNN) for modeling nonlinear systems [11]. This FAOS-PFNN is able to adjust its structure relying on the modeling error in the learning process. The results show that the FAOS-PFNN can obtain a compact structure and high modeling accuracy. Ebadzadeh et al. introduced a new fuzzy network model with correlated fuzzy rules (CFNN) [12]. In CFNN, a correlated structure is used to generate non-separable fuzzy rules for building FNNs. The experimental results indicate that this proposed CFNN can approximate the nonlinear functions better than the others with fewer fuzzy rules. A generalized ellipsoidal basis function based online self-constructing fuzzy neural network (GEBF-OSFNN) was proposed in [13]. The structure of GEBF-OSFNN can be optimized by the training error to improve the approximation and generalization performance. Moreover, a data-driven neural fuzzy system with collaborative fuzzy clustering mechanism (DDNFS-CFCM) was developed for modeling nonlinear systems in [14]. In DDNFS-CFCM, the fuzzy rules are generated by using the fuzzy C-means (FCM) algorithm and adapted by the preprocessed collaborative fuzzy clustering technique. Although these above algorithms [11–14] can design the structure of FNNs in the learning process, they cannot prune the redundant fuzzy rules. To solve this problem, McDonald et al. proposed a self-organizing FNN, which can prune the hidden neurons by using the output intensity of the hidden layer [15]. Rubio et al. introduced a pruning algorithm which can delete the redundant hidden neurons based on the density of fuzzy rules [16]. And a self-evolving neural fuzzy inference networks (SENFIN) was developed in [17]. In SENFIN, a subgroup symbiotic evolution is designed to find the suitable number of fuzzy rules. However,

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these methods [15–17] can only remove the redundant neurons in the learning process. Moreover, another disadvantage of these methods is their heavy computational burden since the majority of the training time is spent on training process which are larger than necessary. Recently, Chen et al. proposed a self-organizing neuro-fuzzy network based on first order effect sensitivity analysis (NFN-FOESA) in [18]. The results show that the NFN-FOESA can find a suitable structure faster than the other methods. And a self-organizing fuzzy-neural-network with adaptive computation algorithm (SOFNN-ACA) was introduced in [19]. This SOFNN-ACA can determine the model size by the high spiking intensities and relative mutual information of fuzzy rules. The modeling results demonstrate that this proposed SOFNN-ACA can model nonlinear systems effectively. Also some other self-organizing FNNs can be found in [20–22]. However, most of these self-organizing FNNs focus more attention on the structure design than the parameters identification.

In order to adjust the parameters of FNNs, the backpropagation (BP) algorithm is considered as one of the most frequently used techniques. As widely reported in the literature, however, the BP algorithm suffers from the excessively long training time and local minimum [23–24]. To improve the learning performance, an accelerated hybrid learning algorithm, combining clustering techniques with an adaptive version of the BP algorithm, was applied for training FNNs [25]. Zhao et al. developed a novel Levenberg–Marquard (LM) optimization method by using an integrated gradient descent learning approach to achieve more accurate approximation [26]. However, a disadvantage of the LM algorithm and its variants is the increased memory requirements caused by calculating the Jacobian matrix of the error function. Moreover, another disadvantage is that the LM algorithm is still a local optimization method. Mashinchi et al. proposed a two phase genetic algorithm (GA)-based learning method [27]. This GA-based FNN can both estimate the optimal fuzzy weights and provide good estimates for the shape of the membership function. The results show that this two phase GA-based learning method is better than the conventional GA-based algorithms. Moreover, Kuo et al. developed a fuzzy neural network based on particle swarm optimization method (IOAP-FNN) in [28]. This IOAP-FNN can determine the relationship between the radio frequency identification signals and the position of a picking cart. The modeling results demonstrate the effectiveness of IOAP-FNN. Furthermore, some other novel evolutionary algorithms have been used to optimize the parameters of FNNs in [29–30]. However, one of the basic disadvantages of the evolutionary algorithms is time-consuming [31–32].

Based on the above analysis, in this study, a self-organizing fuzzy neural network with adaptive gradient algorithm (SOFNN-AGA) is proposed for nonlinear system modeling. This SOFNN-AGA owns high model accuracy with compact network structure and fast convergence in the learning process. The major contributions of this paper are summarized as follows: (1) A novel self-organizing mechanism, based on potentiality of fuzzy rules (PFR), is developed to determine the network size in the learning process. The structure of FNNs can be self-organized by using PFR of each rule. (2) An adaptive gradient algorithm (AGA), with adaptive learning rate, is proposed to adjust the parameters of SOFNN-AGA. This AGA method can improve the learning performance with fast convergence and powerful searching ability. (3) The convergence of SOFNN-AGA has been demonstrated theoretically and experimentally. The theoretical analysis provides some sufficient conditions for convergence.

The remainder paper is organized as follows. Section 2 briefly discusses the basics of FNNs. Section 3 details the proposed SOFNN-AGA, including a novel self-organizing mechanism and the AGA method. Then, the mathematical analysis on the convergence of SOFNN-AGA is given in Section 4. Section 5 reports some experimental results of SOFNN-AGA, which demonstrate some merits in learning speed and modeling accuracy against other existing methods. Finally, the conclusion is given in Section 4.

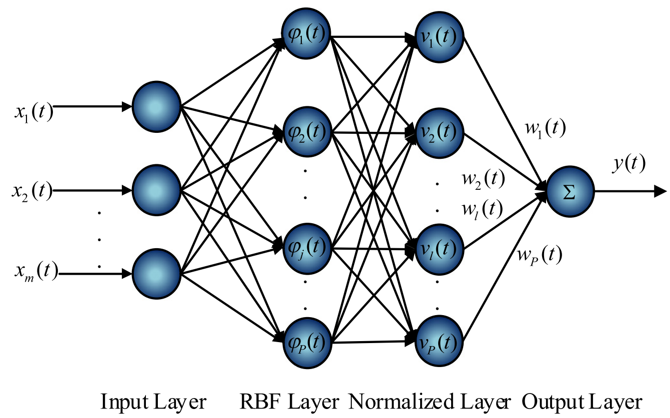


**Fig. 1.** The structure of FNN.

## 2. Problem description

In order to describe SOFNN-AGA clearly, a simple structure of FNN is shown in Fig. 1. The four layers are the input layer, the RBF layer, the normalized layer, and the output layer. A mathematical description of each layer in FNN is given as follows.

*The input layer:* There are $m$ neurons in this layer, which represent the input variables of FNN. The output values of input layer can be expressed as

$$u_i(t) = x_i(t),\ (i = 1, 2, ..., m), \tag{1}$$

where $u_i(t)$ is the $i$th output value at time $t$, and $\mathbf{x}(t)=[x_1(t), x_2(t),..., x_m(t)]$ are the inputs of FNN at time $t$.

*The RBF layer:* Each neuron is a RBF form in this layer. The output values of RBF neurons are

$$\varphi_j(t) = \prod_{i=1}^{m} e^{-\frac{(u_i(t)-c_{ij}(t))^2}{2\sigma_{ij}^2(t)}} = e^{-\sum_{i=1}^{m}\frac{(u_i(t)-c_{ij}(t))^2}{2\sigma_{ij}^2(t)}},$$
$$i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, P, \tag{2}$$

where $\varphi_j(t)$ is the output value of the $j$th neuron at time $t$, $\mathbf{c}_j(t)=[c_{1j}(t), c_{2j}(t),..., c_{mj}(t)]$ is the center of the $j$th RBF neuron at time $t$, and $\boldsymbol{\sigma}_j(t)=[\sigma_{1j}(t), \sigma_{2j}(t),..., \sigma_{mj}(t)]$ is the widths of the $j$th RBF neuron at time $t$, $P$ is the number of neurons in this layer.

*The normalized layer:* There are $P$ neurons in this layer (the number is same as the RBF layer), the output values of this layer are described as

$$v_l(t) = \frac{\varphi_l(t)}{\sum_{j=1}^{P}\varphi_j(t)} = \frac{e^{-\sum_{i=1}^{m}\frac{(u_i(t)-c_{il}(t))^2}{2\sigma_{il}^2(t)}}}{\sum_{j=1}^{P} e^{-\sum_{i=1}^{m}\frac{(u_i(t)-c_{ij}(t))^2}{2\sigma_{ij}^2(t)}}}, \tag{3}$$

where $v_l(t)$ is the $l$th output at time $t$, and $\mathbf{v}(t)=[v_1(t), v_2(t),..., v_P(t)]^{\mathrm{T}}$ is the output vectors at time $t$.

*The output layer:* The output is clarified using the gravity method

$$y(t) = \sum_{l=1}^{P} w_l(t)v_l(t), \tag{4}$$

where $\mathbf{w}(t)= [w_1(t), w_2(t), ..., w_P(t)]$ is the output weights, $w_l(t)$ is the weight between the output layer and the $l$th normalized neuron at time $t$, and $y(t)$ is the output of FNN at time $t$.

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