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Nonconvex function activated zeroing neural network models for dynamic quadratic programming subject to equality and inequality constraints☆

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A B S T R A C T

Zeroing neural network (ZNN, or termed Zhang neural network after its inventor), being a special type of neurodynamic methodology, has shown powerful abilities to solve a great variety of time-varying problems with monotonically increasing odd activation functions. However, the existing results on ZNN cannot handle the inequality constraint in the optimization problem and nonconvex function cannot applied to accelerating the convergence speed of ZNN. This work breaks these limitations by proposing ZNN models, allowing nonconvex sets for projection operations in activation functions and incorporating new techniques for handing inequality constraint arising in optimizations. Theoretical analyses reveal that the proposed ZNN models are of global stability with timely convergence. Finally, illustrative simulation examples are provided and analyzed to substantiate the efficacy and superiority of the proposed ZNN models for real-time dynamic quadratic programming subject to equality and inequality constraints.

1. Introduction

In addition to the remarkable features such as parallelism, distributed storage and adaptive self-learning capability, neural networks can be readily implemented by hardware, and have thus been applied widely in many fields [1–7]. For example, Ahmad et al. design a model based on neural network in [8] to solve the higher order nonlinear boundary value problems. They further develop different models based on neural network to solve various problems in [9–11]. Zeroing neural network as well as its variant (i.e., zeroing dynamic), as a systematic approach to online solution of time-varying problems with scalar situation included [12–16], has been applied to the online matrix inversion [17], motion generation and control of redundant robot manipulators [18,19], tracking control of nonlinear chaotic systems [20] or even the control of populations in mathematical ecology [21]. For example, a ZNN model with a nonlinear function activated is applied to the kinematic control of redundant robot manipulators via Jacobian matrix pseudoinversion in [12], which achieves high accuracy but cannot handle the bound constraints existing in the robots. Miao et al. present a finite-time convergent ZNN model for solving dynamic quadratic programs with application to robot tracking, which requires convex activation functions and cannot remedy the issue of joint-limit avoidance. Such a ZNN method is further discretized to compute the solution to time-varying nonlinear equations based on a new three-step formula in [13], which can be implemented on digital computer directly. In addition, for the applications, ZNN is exploited in [18] to remedy the joint-angle drift phenomenon of redundant robot manipulators by minimizing the difference between the desired joint position and the actual one.

It is worth pointing out here that, although these existing models differ in choosing different error functions or using different activation functions, all of them follow similar design procedures: the ZNN method usually formulates a time-varying problem into a

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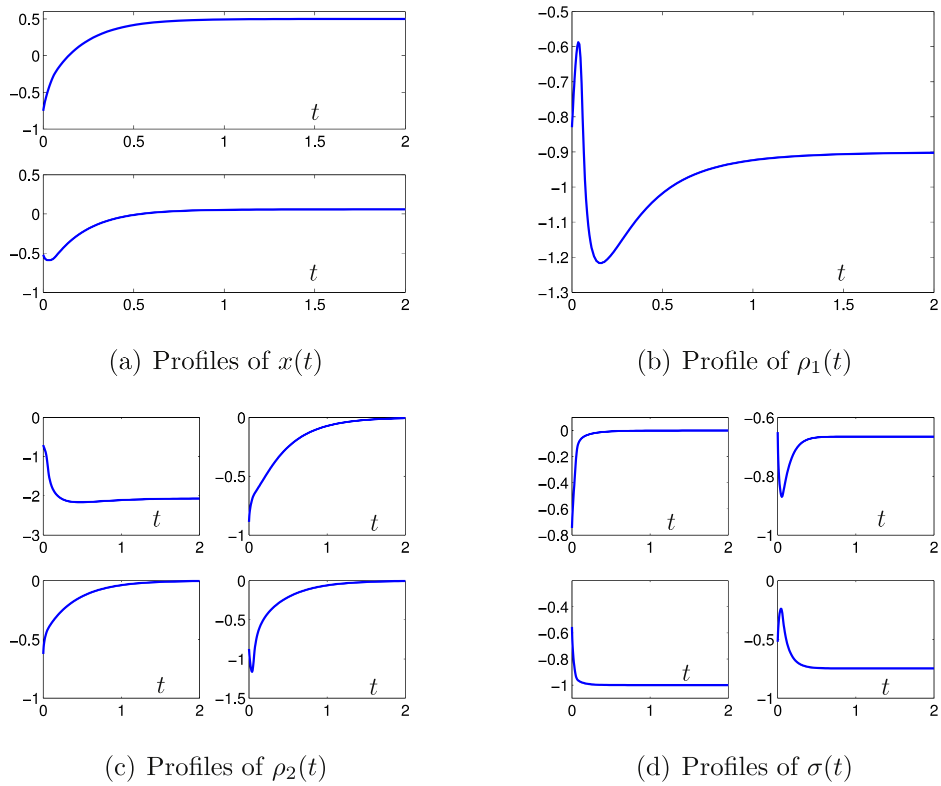

Fig. 1. State vector $y(t) = [x(t), \rho_1(t), \rho_2(t), \sigma(t)]^T$ of ZNN model (8) for solving (13) at $t = 1$.

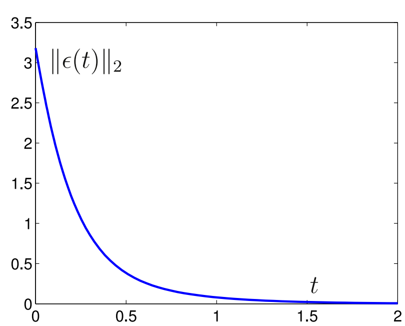

Fig. 2. Residual error of ZNN model (8) for solving (13) at $t = 1$.

Table 1
Comparisons among different models for dynamic problems, where AF denotes activation functions.

	Nonconvex Ω Allowed for AF	Inequality constraints allowed	Theoretical error	Bound constraints allowed
Model in [12]	No	No	Zero	No
Model in [15]	No	No	Zero	No
Model in [16]	No	NA*	Nonzero	No
Model in [22]	No	No	Zero	No
Models in [23]	No	No	Nonzero	No
Model I in this paper	No	Yes	Zero	Yes
Model II in this paper	Yes	Yes	Zero	Yes

*Note that the model in [16] is presented for dynamic matrix inversion and NA means that the item does not apply to the model.

regulation problem in control. Specifically, the residual error of a ZNN model for the task function to be solved is to be regulated to zero. Then, a monotonically increasing and odd function activated ZNN model with its equilibrium identical to the solution of this time-varying problem is devised to solve the latter recursively. In addition, the design parameter in the ZNN method should be larger than 0. To the best of the authors' knowledge, all existing results on ZNN assume that the set for projection of activation function is a convex one, which evidently excludes nonconvex set from consideration. General conclusions relaxing the convex constraint on activation functions remain unexplored.

In this paper, we make progress along this direction by proposing new results on ZNN to remedy these weaknesses. As summarized in Table 1, the proposed ZNN models in this paper are able to deal with nonconvex projection set Ω in the activation functions, while existing ones [12,15,16,22,23] require the projection set to be convex. Additionally, this is the first work on ZNN for solving a time-varying optimization problem with inequality and bound constraints, which opens a door to the research on solving time-varying constrained optimization problems in an error-free manner. In short, there are two limitations in the existing researches on ZNN, i.e., lacking the technique for handling inequality and bound constraints when solving dynamic optimization problems and requiring the activation function to be odd and monotonically increasing. This work breaks these limitations by proposing ZNN models, allowing nonconvex sets for projection operations in activation functions and incorporating new techniques for handing inequality constraint. The remainder of this paper is organized into four sections. The problem formulation and the new ZNN models for dynamic quadratic programming subject to equality and inequality constraints are presented in Section 2. Then, theoretical analyses are presented in Section 3. Section 4 provides illustrative simulation examples to substantiate the efficacy and superiority of the proposed ZNN models for dynamic quadratic programming. Section 5 concludes the paper with final remarks. Before ending this introductory section, the main contributions of this paper are listed as follows:

(1) This paper focuses on solving dynamic quadratic programming subject to equality and inequality constraints with time-varying

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