Applied Soft Computing 46 (2016) 792–804

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

Image smoothing with generalized random walks: Algorithm and applications

Zhaobin Wang*, Hao Wang

School of Information Science and Engineering, Lanzhou University, Lanzhou 730000, China

ARTICLE INFO

Article history:
Received 28 May 2015
Received in revised form
18 December 2015
Accepted 5 January 2016
Available online 12 January 2016

Keywords:
Edge detection
Image decomposition
Image enhancement
Image smoothing
Random walks

ABSTRACT

A novel generalized random walks model based algorithm for image smoothing is presented. Unlike previous image smoothing methods, the proposed method performs image smoothing in a global weighted way based on graph notation, which can preserve important features and edges as much as possible. Based on the new random walks model, input image information and user defined smoothing scale information are projected to a graph, our method calculates the probability that a random walker starting at each pixel node position will first reach one of the pre-defined terminal node to achieve image smoothing, which goes to solving a system of linear equations, the system can be solved efficiently by lots of methods. Theoretical analysis and experimental results are reported to illustrate the usefulness and potential applicability of our algorithm on various computer vision fields, including image enhancement, edge detection, image decomposition, high dynamic range (HDR) image tone mapping and other applications.

1. Introduction

Image smoothing is one of the most fundamental and widely studied problem in computer vision. Image smoothing, also called image blurring. Traditional smoothing methods not only dissolve the noise, but also blur the important edges. To avoid this, edge preserving smoothing is proposed. Edge preserving smoothing method may be viewed as a compromise between smoothing and edge preserving. It wipes out tiny details and noise, and at the same time, preserves salient important edges. In the past few years, it has been successfully applied in the field of image processing such as edge detection, image restoration, image enhancement, and many other high-level image processing tasks. As an integral step of many computer vision problems, the results of image smoothing influence the performance of the whole vision system.

Our work is partially inspired by the well-known graph cut model [1], in this paper we propose a novel approach to edge preserving image smoothing with a similar graph structure. However, theoretical foundation of our work is based on random walks algorithm [2,3], the random walks algorithm is used for image segmentation by Grady, it solves this problem as following: given a random walker starting at each pixel position, what is the probability that it first reaches each of the pre-labeled nodes. A generalized random walks model for image smoothing is proposed. The smoothing problem is formulated on a carefully designed graph. First an image model is constructed, an input image is treated as a graph with a fixed number of vertices and edges. Each edge is assigned a positive value weight corresponding to image gradient. It indicates the likelihood that a random walker will cross that edge, which is the same as traditional random walks algorithm. Besides, two auxiliary nodes are added to the model and each pixel node is connected with the two nodes. Edges between the two nodes and pixel nodes are also assigned a weight to indicate that random walker will reach a terminal node from pixel node. By calculating the probability that the random walker first reaches one of the terminal nodes from each pixel node position, the smoothing result can be obtained.

The main contribution of the paper is to propose a new strategy that can project information of image structure and smoothing willing to a graph model. Our approach gives a new understanding of how edge preserving smoothing can be realized, which can fully explain the reason why image smoothing problem can be solved by random walks algorithm; Algorithmically, we propose a system of linear equations in an optimization framework for image smoothing and apply it to some interesting applications; The relationship between our method and generalized random walk with restart [4,5], anisotropic diffusion [6] and WLS [7] is also discussed.

The rest of the paper is organized as follows. Section 2 reviews previous methods. Differences between our algorithm and some existing image smoothing methods are also discussed. Section 3 provides implementation details of our method and theoretical connections with previous related work. Section 4 discusses experimental results and performance, along with comparisons with other smoothing methods. Finally, Section 5 gives the conclusions.

2. Prior work

Our work benefits from the rich body of work on image smoothing and random walk based applications. Some related work is reviewed in this section.

2.1. Prior image smoothing methods

Anisotropic diffusion model introduced by Perona and Malik is widely used in practice and extensively studied in theory [6], it is modeled using partial differential equations and implemented as an iterative process, it uses an edge-stopping function of local

* Corresponding author. Tel.: +86 9318912786.
E-mail address: zhaobin_wang@hotmail.com (Z. Wang).

http://dx.doi.org/10.1016/j.asoc.2016.01.003
1568-4946/

gradient to make smoothing take place only in the interior of regions without crossing edges. It has been demonstrated to be able to achieve a good trade-off between noise removal and edge preservation. There has been a large number of works aimed at optimizing and extending the idea [8,9]. Rudin et al. proposed to regularize total variation, which utilizes the gradient sparsity enforced by an L1 penalty term to do edge preserving smoothing [10]. It has been successful in denoising problems. It is the inspiration source of much work. By now, the algorithm has also been used for other restoration tasks such as deblurring, blind deconvolution and inpainting [11]. The bilateral filter computes a smoothed output with a weighted average of neighboring pixels intensities, it takes into account spatial and color distances [12]. It is an extension of typical Gaussian smoothing. There exist a number of methods to boost the algorithm [12,13]. Because of its simplicity and effectiveness, it is one of the most popular smoothing methods.

In recent years, many exciting new techniques have been invented to solve this problem. All the methods have demonstrated satisfying results with good performance. Farbman et al. proposed to perform the edge preserving smoothing using the weighted least square (WLS) framework [7,14], edge preserving smoothing is viewed as a compromise between data term and regularization term. By minimizing the proposed energy functional, image smoothing result can be obtained by solving a large linear system. Xu et al. presented L0 gradient minimization for image smoothing, it minimizes a specific objective function [15], it can remove low-amplitude structures and globally preserve and enhance salient edges. Subr et al. considered edge preserving smoothing as interpolation between local signal extremes [16]. The method defines detail as oscillations between local minima and maxima. It smoothes high contrast texture while preserving salient edges. Guided image filter assumes there is a local linear model between the guidance image and the filtering output, and they seek a solution that minimizes the difference between filtering input and output while maintaining the linear model [17].

Among different smoothing schemes, our proposed graph theory based method has several good features in practical applications. It explicitly organizes the image elements into mathematical structures, requires no discretization and therefore incurs no discretization errors or ambiguities, and makes the formulation of the problem more flexible and the computation more efficient [3].

2.2. Random walks

The term random walk was first proposed by Karl Pearson in 1905 [19]. In his letter to nature, Pearson proposed the problem of random walk. At each step, a man started from a point and moved a fixed length, with a randomly chosen angle. He wanted to know the distribution of the man after many steps had been taken. In the same year, Albert Einstein published his paper on Brownian motion which he modeled as a random walk [20]. It had an enormous impact, and it gave strong evidence for discrete particles at a time when most scientists believed that matter was a continuum. The concept of random walk has been used in many fields, and it is nearly ubiquitous in science and engineering, including ecology, economics, physics, chemistry, and biology [20,22,23].

Various properties of random walks can be used to define the specified algorithm. The average first-passage time which is defined as the average number of steps a random walker starting in a state will take to enter another state for the first time; the average commute time which is defined as the average number of steps a random walker starting in a state will take before entering another state for the first time and go back to starting state; the escape probability which is defined as the probability that a random walk starting at a state will reach another state before returning to starting state, they are applied in many applications [23–27].

Random walks proposed by Grady are an improvement to traditional random walks on graph by highlighting the diversity and centrality simultaneously [3]. It obtains the affinities between each node and labeled nodes by measuring probability that a random walker will first reach a labeled node before other labeled nodes. The probability problem shares the same solution as the harmonic functions with given boundary conditions. The harmonic function defined on graph can be easily solved by minimizing a combinatorial formulation of the Dirichlet integral, and through certain mathematical transformations, the final question goes to solving a system of linear equations. Now the algorithm has been successfully applied in many fields of image processing, e.g. image segmentation, image fusion, image annotation and classification, 2D–3D conversion and other applications [2,29,30].

3. Proposed smoothing method

3.1. Problem formulation

Unlike most previous image smoothing methods, we consider image smoothing as a problem: given a random walker starting at each pixel node position, what is the probability that he first reaches one of the two terminal nodes? As illustrated in Fig. 1, a walker starting from an image pixel node position, he may go directly to the terminal nodes or he may wander around his starting position before he reaches terminal nodes. At last, he will reach one of the terminal nodes before another. Our method calculates the probability that he will end up with reaching each terminal nodes, and takes each probability starting from each pixel positions as smoothed pixels intensities. The process is explained in detail below.

Suppose an input image I with n pixels is scaled to [01], image I can be written in a vector form $[I_1, I_2, \ldots, I_n]^T$. Image pixel lattices are represented in the form of a weighted undirected graph $G=(V, E, W)$, where V is a set of vertices $\{1, 2, \ldots, n\}$, represents image pixels with intensities $\{I_1, \ldots, I_n\}$, E is a set of edges $\{e_{ij}\}$ connecting two neighboring pixels I_i and I_j, W is a weighted adjacency matrix of input image, whose elements $\{w_{ij}\}$ are assigned to each edge e_{ij}. In this work, we use the typical Gaussian weighting function to calculate the weight given by

$$w_{ij} = e^{-\beta(I_i - I_j)^2}, \quad (1)$$

where I_i and I_j indicate the image intensity at pixel i and j, and β is a free parameter. A big weight value means a close relationship.

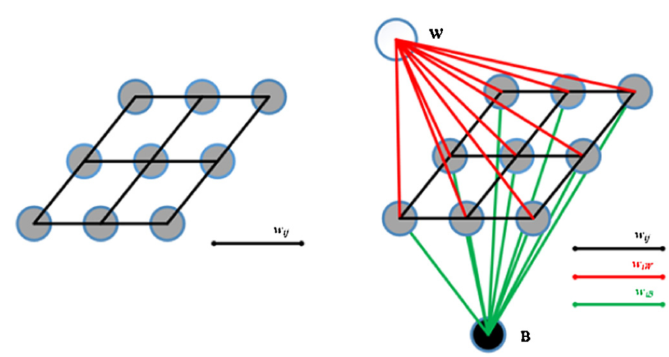

Fig. 1. Traditional random walk model (left) and the proposed model (right) for image smoothing (3 × 3 image). On the traditional graph model, each pixel is connected with its four neighbors (if it has). We added some other vertex and edges in our proposed graph model for image smoothing. Terminal nodes **W** and **B** are added and further connected with each pixel node. Random walkers starting from each pixel node position can walk to its surrounding pixel node position through the edge, in the long term walking process, he will wander among the pixels nodes before reaching terminal nodes. By calculating the probability walkers will first arrive terminal node **W** before **B**, we get our image smoothing result. Section 3 gives detailed instructions.