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Neural-network-based adaptive guaranteed cost control of nonlinear dynamical systems with matched uncertainties☆

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ABSTRACT

In this paper, we investigate the neural-network-based adaptive guaranteed cost control for continuous-time affine nonlinear systems with dynamical uncertainties. Through theoretical analysis, the guaranteed cost control problem is transformed into designing an optimal controller of the associated nominal system with a newly defined cost function. The approach of adaptive dynamic programming (ADP) is involved to implement the guaranteed cost control strategy with the neural network approximation. The stability of the closed-loop system with the guaranteed cost control law, the convergence of the critic network weights and the approximate boundary of the guaranteed cost control law are all analyzed. Two simulation examples have been conducted and all simulation results have indicated the good performance of the developed guaranteed cost control strategy.

1. Introduction

During control practices, the design on nonlinear optimal regulators always involves to solve the Hamiltonian–Jacobi–Bellman (HJB) equation. Although the nonlinear optimal problem is well described by the HJB equation from the point of view of mathematics, it is well known that the analytical solution of nonlinear HJB equation is almost impossible to be obtained since the problem itself encounters the partial differential equation. The approach of dynamic programming was deemed as a basic strategy to handle optimal control problems of nonlinear systems, but there still exists a serious issue called the curse of dimensionality [1,2]. To overcome the difficulty in coping with optimal control problems of nonlinear systems, based on function approximation structures like neural networks and support vector machines, approximate or adaptive dynamic programming (ADP) was proposed as a kind of effective method to solve nonlinear optimal control problems in forward time [3–8]. There exists a fundamental idea in ADP which is similar as adaptive control systems with function approximation techniques [9–13].

ADP and relevant methods have gained much development in various control issues of nonlinear systems. In [14], the control-constrained approximate optimal control was proposed for discrete-time nonlinear systems by the finite-horizon non-quadratic cost function based on an adaptive critic network structure. In [15], the optimal control problem of unknown and non-affine nonlinear systems was addressed by the ADP-based method and recurrent neural networks. A novel policy iteration method of global ADP was proposed in [16] for the adaptive optimal control of nonlinear polynomial systems. The H_∞ state feedback control problem for a class of affine nonlinear discrete-time systems was investigated in [17] with unknown system dynamics. More topics of nonlinear systems have also been conducted with the ADP-based approach, such as optimal control of discrete-time nonlinear systems [18–21], optimal control of continuous-time nonlinear systems [22–24], optimal tracking control [25–27], robust control [28–30], differential games [31–33], and so on.

It is worth mentioning that the researchers have paid great attention on system uncertainties since uncertain systems exist in a broad rage [34–37]. For example, in [34], the robust filtering problem of the stochastic systems with polytopic uncertainties was investigated. [35] presented the state estimation for complex networks with uncertain inner coupling. In [36], the robust stabilization of uncertain switched neutral systems was intensively developed based on Lyapunov stability theory and the dynamic

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output feedback technique. In [37], a robust output observer-based control was proposed for the problem of uncertainty in switched neutral systems with interval time-varying mixed delays. Simultaneously, the optimal control design on uncertain systems is more and more frequently consulted. As is known to all, the direct optimal control design of uncertain nonlinear systems is pretty difficult, since coping with the cost function of the uncertain system is not an easy task. Therefore, some researchers have paid attention to study the boundedness of the cost function with respect to the uncertain system, not limited to directly optimize it. The guaranteed cost control strategy, proposed by Chang and Peng [38], possesses the advantage of providing an upper bound on a given cost function, and therefore the cost function can be limited within this boundary even if the degradation of control performance is incurred by system uncertainties.

When the boundedness of the cost function of the uncertain systems is considered, it results in the guaranteed cost control problem. There are some results in the field of the guaranteed cost control design with the ADP-based approach. In [39], the ADP-based guaranteed cost control was firstly proposed for continuous-time uncertain nonlinear systems, and a modified cost function was established as the guaranteed cost function of the studied uncertain system. In [40], for a class of time-varying uncertain nonlinear systems, a neural-network-based approximate optimal guaranteed cost controller was developed with respect to a finite-horizon cost function. In [41], an ADP-based guaranteed cost control algorithm was presented for the tracking control of continuous-time uncertain nonlinear systems. In [42], the decentralized guaranteed cost control design was constructed for large-scale uncertain nonlinear systems, where both dynamical uncertainties and interconnections were discussed for better control performance. Using the ADP-based approach, an available guaranteed cost control scheme can be obtained for nonlinear system by approximately solving a optimal control problem. Most of the existing results of this field are obtained in terms of optimal regulation or tracking problems [39–42], not the guaranteed cost control problem regarding to unknown system dynamics. This greatly motivates our research.

This paper mainly contributes to the neural-network-based guaranteed cost control for continuous-time uncertain nonlinear systems with unknown dynamics. First, the cost functions of the original uncertain system and its associated nominal system are both defined. It is proved that the optimal control of the associated nominal system can implement the guaranteed cost control for the original uncertain system. Second, a neural network identifier is involved to the guaranteed cost control scheme. The weights of neural network identifier are adaptively updated to ensure the asymptotical stability of the identification error. Third, with the ADP-based technology, a novel learning control framework is built to approximately solve the optimal control of the nominal system for implementing the guaranteed cost control law of the original uncertain system. Theoretical analysis is provided for the stability of the closed-loop system with the learning-based guaranteed cost control, as well as the approximate boundary of the guaranteed cost control law.

The rest of this paper is organized as follows. The problem description of the guaranteed cost control for a class of continuous-time affine nonlinear systems is provided in Section 2, and the problem transformation is also stated. The adaptive critic methodology within the ADP framework is developed for the guaranteed cost control design in Section 3, and the associated theoretical analysis is also presented in this section. The simulation studies on two nonlinear systems are shown in Section 4. Finally, the conclusion remarks are given in Section 5.

2. Problem statement of the guaranteed cost control

We study a class of continuous-time affine nonlinear systems with the formula

$$\dot{x}(t) = f(x) + g(x)u(t) + \Delta f(x), \tag{1}$$

where $x(t) \in \Omega_x \subseteq \mathbb{R}^n$ is the state vector and $u(t) \in \Omega_u \subseteq \mathbb{R}^m$ is the control vector, $f(x)$ and $g(x)$ are differentiable in their arguments with $f(0) = 0$ and $\|g(x)\| \le g_M$, g_M is a positive constant. $\Delta f(x)$ denotes the uncertain dynamics. In this paper, we consider that the uncertain dynamics satisfies the matching condition, i.e., it is in the range space of $g(x)$ rendering $\Delta f(x) = g(x)d(x)$ with $d(x) \in \mathbb{R}^m$. Besides, assume that $d(x)$ is upper bounded by a known function $D(x)$, i.e., $\|d(x)\| \le D(x)$ with $D(0) = 0$. Here, we also assume that $d(0) = 0$ such that $x = 0$ is an equilibrium of system (1).

For the uncertain nonlinear system (1), consider the cost function given as

$$J(x, u) = \int_t^{\infty} R(x(\tau), u(\tau))\mathrm{d}\tau, \tag{2}$$

where $R(x, u) = Q(x) + u^T u$ is the utility function, and $Q(x) = x^T Q x$ with $Q = Q^T \ge 0$.

The purpose of designing the guaranteed cost controller is to find a feedback control function $u(x)$ and determine a finite upper bound function $\Phi(u)$ (where $\Phi(u) \le M < +\infty$ with M being a positive constant), such that the closed-loop system (1) is robustly stable and the cost function (2) is bounded as $J(x, u) \le \Phi(u)$. Therefore, the function $\Phi(u)$ can effectively bound the cost function of system (1), named as the guaranteed cost function. Furthermore, when $\Phi(u)$ is minimal, it is termed as the optimal guaranteed cost and is written as Φ^*, where $\Phi^* = \min_u \Phi(u)$. The associated control law u^* is called the optimal guaranteed cost control law with $u^* = \arg\min_u \Phi(u)$. In this paper, we will study how to obtain the optimal guaranteed cost control law u^* for system (1) with the cost function (2).

Without considering the uncertainty, the controlled system turns to its nominal version, which plays an important role in the control design. The nominal system corresponding to (1) is formulated as

$$\dot{x}(t) = f(x) + g(x)u(t). \tag{3}$$

Similar as the classical literature of nonlinear optimal control, assume that the right side of (3) is Lipschitz continuous on a set $\Omega_x \subseteq \mathbb{R}^n$ containing the origin such that system (3) is controllable.

The following conclusions present the achievement of nonlinear robust control and the existence of guaranteed cost function of system (1), as an improvement of the result of [39]. Note that $\nabla(\cdot) \triangleq \partial(\cdot)/\partial x$ is employed to denote the gradient operator.

Theorem 1. *There exists a continuously differentiable function $V(x)$ for system (1) with $V(x) \ge 0$ and $V(0) = 0$ only at $x = 0$, such that for a feedback control law $u(x)$ satisfying*

$$(\nabla V(x))^T g(x) = -2u^T(x), \tag{4a}$$

$$R(x, u) + (\nabla V(x))^T (f(x) + g(x)u(x)) + \theta D^2(x) = 0, \tag{4b}$$

where $\theta \ge 1$. Then system (1) is locally asymptotically stable within the neighbourhood of the origin under the control law $u(x)$.

Proof. Define $V(x)$ as the Lyapunov function and let $\dot{V}(x)$ be its derivative along system (1). By using (4), we can find that

$$\begin{aligned}\dot{V}(x) &= (\nabla V(x))^T (f(x) + g(x)u(x) + \Delta f(x)) \\ &= -\theta D^2(x) - R(x, u) - 2u^T(x)d(x).\end{aligned} \tag{5}$$