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Distributed cubature information filtering based on weighted average consensus

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ABSTRACT

In this paper, the distributed state estimation (DSE) problem for a class of discrete-time nonlinear systems over sensor networks is investigated. First, based on weighted average consensus, a new DSE algorithm named *distributed cubature information filtering* (DCIF) algorithm is developed to address the high-dimensional nonlinear DSE problem. The proposed filtering algorithm not only has such advantages as easy initialization and less computation burden, but also possesses the guaranteed stability regardless of consensus steps. Moreover, it is proved that the corresponding estimation is consistent, and its mean-squared estimation errors are exponentially bounded. Finally, numerical simulations are given to verify the effectiveness of DCIF.

1. Introduction

With broad applications of large-scale sensor networks in the fields such as target tracking, environment monitoring and wireless camera networking, the study of distributed state estimation (DSE) in various sensor networks has risen to a plethora of popularities due to its distinct advantages over most of the centralized estimation techniques [1]. The major advantages of DSE lie in scalability, low communication burden, and robustness to individual sensor failures [2,3]. During the past two decades, many techniques have been developed to address a variety of problems related to DSE (see, e.g., [4–21]). Among the existing techniques, the consensus-based methodology is the most popular one. For example, with respect to linear discrete-time Gaussian systems, Olfati-Saber et al. have proposed the distributed Kalman filtering (DKF) algorithm by exploiting average consensus algorithms [7–9]. Furthermore, based on the extended Kalman filtering (EKF) algorithm, DKF algorithm is directly extended to nonlinear Gaussian systems, which yields the distributed extended Kalman filtering (DEKF) algorithm [10]. However, DEKF algorithm has inherent drawbacks such as instability and low-order accuracy when systems experience high nonlinearities. Compared with EKF algorithm, the unscented Kalman filtering (UKF) algorithm turns out to be of higher stability robustness and better estimation accuracy. By exploiting a statistical linear regression approach and reconstructing a pseudo measurement matrix, Li and Jia have presented a distributed UKF algorithm for jump Markov nonlinear systems [11]. Lately, without approximating any pseudo measurement matrices, a weighted average consensus-based UKF algorithm has been developed in [12].

A crucial limitation about the UKF-like techniques is that the non-positive definite covariance matrix may arise, especially when systems are high-dimensional [22,23]. To overcome this, the cubature Kalman filter (CKF) is proposed for the high-dimensional nonlinear state estimation [22]. Furthermore, the cubature information filter (CIF), an algebraical equivalence of CKF, is developed in [24]. More precisely, CKF is a Gaussian approximation of a Bayesian filter, but provides estimations more precise and stable than most existing Gaussian filters [24]. Thus, it is no wonder that CKF has been broadly studied in various settings [25–27]. Recently, with rapid developments of sensor technologies and increasing demands of large-scale sensor networking, researchers have turned their attention to designing the distributed CKF (or simply, DCKF) in networked environments [13–17]. Unfortunately, several consensus filtering problems encountered in the DCKF setting have not been examined satisfactorily despite their practical importance.

In general, the consensus-based DSE approaches can be classified into three groups. The first group is consensus on estimates

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(CE), which performs an average consensus on local state estimates [7]. In this view, the work of [13] belongs to this group. However, CE involves no error covariance matrices. As is known, error covariance matrices contain information useful to improve the filter performance. The second group is consensus on measurements (CM) [8,9], which performs consensus on the local innovation parts [12]. It is shown that when the number of consensus steps during each sampling interval is sufficiently large, CM can approximate the correction step of the centralized Kalman-like filter. In this sense, the works [14–16] fall into this group; more specifically, these techniques adopt different forms of CKF to achieve CM. Indeed, CIF is used in [14], while the square root CIF is employed in [15,16] to avoid numerically sensitive matrix operations. The third group falls into consensus on information (CI) [28]. From an algorithm viewpoint, CI is nothing but reaching a local average on information vectors and information matrices. Stability of CI algorithms can be guaranteed for any number of consensus steps (even a single one). As mentioned above, the achievements in the DCKF setting have been obtained either in the CE or CM paradigm. In this paper, we focus our attention on embedding the CI architecture into DCKF to extract possible benefits from the former's positive features. The resulting algorithm is named the distributed cubature information filtering (DCIF) algorithm.

A fundamental property of an estimator is consistency [29], which is significant for information fusion over sensor networks. When fusing information with unknown correlations, simply neglecting the unknown correlations may cause inconsistency in estimation [30]. Ref. [28] has reported results about consistency analysis. More recently, the features about consistency have also been investigated in [30]. However, the results about consistency analysis have been achieved only in the linear setting. Hence, it is also our goal to give a rigorous proof to the consistency of our proposed DCIF algorithm but in the nonlinear setting. By constructing a collective Lyapunov function, it has been shown that under network connectivity and collective observability, the distributed EKF algorithm in [31] can achieve local stability. With a different viewpoint, the stochastic boundedness of estimation errors for a class of consensus-based UKF algorithms has been verified by means of the stochastic stability lemma in [12]. However, stochastic stability analysis remains unsolved in the DCKF setting.

Motivated by the above researches, we explicate our proposed DCIF algorithm by exploiting a weighted average consensus approach. Furthermore, we attempt to analyze the consistency of the proposed DCIF algorithm, while boundedness analysis of estimation errors is attacked. The main contributions include: (1) a more accurate and stable distributed nonlinear filtering algorithm is well-developed, which applies to a wide range (from low to high dimensions) of nonlinear DSE problems; (2) by deriving a pseudo system matrix and a pseudo measurement matrix, the consistency of estimates for a class of consensus-based CIF algorithms is proven; (3) by means of the stochastic stability lemma, the stochastic boundedness of estimation errors for the proposed DCIF algorithm is investigated.

The outline is as follows. Section 2 models the sensor network in nonlinear dynamic systems. The general CIF algorithm is presented in Section 3. Section 4 develops our proposed DCIF algorithm, while its stability analysis is presented in Section 5. Furthermore, Section 6 illustrates numerical simulations. Finally, Section 7 concludes this paper.

Throughout the paper, we write $XX^T = X(*)^T$, $X^TYX = (*)^TYX$ and $XYX^T = XY(*)^T$ to save space. $\mathbb{R}^n$ represents the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ represents the set of all $n \times m$ real matrices. $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^n$. I_n is the $n \times n$ identity matrix and $\mathrm{diag}(B_1, B_2, \ldots, B_n)$ refers to a diagonal matrix with its main diagonal matrix block being $B_1, B_2, \ldots, B_n$. For an arbitrary matrix A, A^T and A^{-1} denote its transpose and inverse, respectively; $\mathrm{tr}\{A\}$ represents the trace of A and $A > 0$ means A is a positive define matrix. $\mathbf{E}\{\cdot\}$ denotes the expectation operation. $\mathbf{E}^{-1}\{\cdot\} = (\mathbf{E}\{\cdot\})^{-1}$ is used for brevity.

2. Sensor network modeling

Consider a discrete-time dynamic system with an N-sensor network (N is the total number of sensor nodes; in the case of $N \geq 2$, the sensors are located in a distributed fashion), which possesses collectively the following discrete-time nonlinear system subject to additive Gaussian noise:

$$\begin{cases} x_k = f(x_{k-1}) + \omega_{k-1} \\ z_k^s = h^s(x_k) + \nu_k^s, \quad s = 1, 2, \ldots, N \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the state vector of the dynamic system at discrete-time instant k. $z_k^s \in \mathbb{R}^r$ represents the measurement vector of the sth sensor. The process noise $\omega_{k-1} \in \mathbb{R}^n$ and the measurement noise $\nu_k^s \in \mathbb{R}^r$ are uncorrelated zero-mean Gaussian white sequences with covariance matrices $Q_{k-1} \in \mathbb{R}^{n\times n}$ and $R_k^s \in \mathbb{R}^{r\times r}$, respectively. The first and second equations in (1) are the process equation and the measurement equation, respectively. $f : \mathbb{R}^n \to \mathbb{R}^n$ describes the nonlinear state transition function and $h^s : \mathbb{R}^n \to \mathbb{R}^r$ describes the nonlinear measurement function of the sth sensor. These functions are assumed to be known. The communication topology of the sensor network is denoted by an undirected graph $\mathcal{G}(\mathcal{N}, \mathcal{E})$, where $\mathcal{N} = \{1, 2, \ldots, N\}$ is the sensor node set and $\mathcal{E}$ is the edge set. An edge $(s, j) \in \mathcal{E}$ means that node j can receive data from node s in its neighbors and vice versa. For each node $s \in \mathcal{N}$, $\mathcal{N}_s = \{j|(j, s) \in \mathcal{E}\}$ denotes the set of its neighbors; if node s is included at least in one of its neighbors' sets, we write $\mathcal{J}_s = \mathcal{N}_s \cup \{s\} \neq \emptyset$.

3. Cubature information filtering algorithms

For each sensor node s, the general CIF algorithm is summarized as follows, which is a two stage procedure containing time update and measurement update.

(1) *Time update:* Let m $(= 2n)$ cubature points $\chi_{k-1|k-1}^{s,i} \in \mathbb{R}^n$ be generated based on the state estimate $\hat{x}_{k-1|k-1}^s$ and the square-root matrix $S_{k-1|k-1}^s$ at time step $k-1$.

$$\chi_{k-1|k-1}^{s,i} = S_{k-1|k-1}^s \xi_i + \hat{x}_{k-1|k-1}^s, \quad i = 1, \ldots, m \tag{2}$$

where

$$\xi_i = \begin{cases} \sqrt{n} e_i, & 1 \leq i \leq n \\ -\sqrt{n} e_{i-n}, & n+1 \leq i \leq m. \end{cases} \tag{3}$$

e_i is the n-dimensional unit vector with the ith element being 1 and $S_{k-1|k-1}^s$ is the square-root matrix of $(Y_{k-1|k-1}^s)^{-1}$. Then, each cubature point $\chi_{k-1|k-1}^{s,i}$ is mapped through the nonlinear state transition function $f(\cdot)$ as

$$\chi_{k|k-1}^{*s,i} = f(\chi_{k-1|k-1}^{s,i}) \in \mathbb{R}^n, \quad i = 1, \ldots, m. \tag{4}$$

Next, the predicted state $\hat{x}_{k|k-1}^s$, the predicted information matrix $Y_{k|k-1}^s$ and the predicted information state vector $\hat{y}_{k|k-1}^s$ are determined by

$$\begin{cases} \hat{x}_{k|k-1}^s = \frac{1}{m} \sum_{i=1}^m \chi_{k|k-1}^{*s,i} \in \mathbb{R}^n \\ Y_{k|k-1}^s = [\frac{1}{m} \sum_{i=1}^m \chi_{k|k-1}^{*s,i} (*)^T - \hat{x}_{k|k-1}^s (*)^T + Q_{k-1}]^{-1} \in \mathbb{R}^{n\times n} \\ \hat{y}_{k|k-1}^s = Y_{k|k-1}^s \hat{x}_{k|k-1}^s \in \mathbb{R}^n. \end{cases} \tag{5}$$

(2) *Measurement update:* Firstly, another set of new cubature points $\chi_{k|k-1}^{s,i} \in \mathbb{R}^n$ are produced based on the predicted state $\hat{x}_{k|k-1}^s$