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A new distance measure for interval valued intuitionistic fuzzy sets and its application to group decision making problems with incomplete weights information

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ABSTRACT

The aim of this study is to introduce a novel generalized distance measure for interval valued intuitionistic fuzzy sets and to illustrate the applicability of the proposed distance measure to group decision making problems. Firstly, a generalized distance measure is proposed along with proofs satisfying its axioms. Then, a comparison between the proposed distance measure and well-known distance measures is performed in terms of counter-intuitive cases. Subsequently, the extension of TOPSIS method, in which the proposed distance measure is used to calculate separation measures, to an interval valued intuitionistic fuzzy (IVIF) environment is demonstrated to solve multi-criteria group decision making (MCGDM) problems using optimal criteria weights determined with linear programming model based on the concept of maximizing relative closeness coefficient. Finally, two illustrative examples are provided for proof-of-concept purposes and to demonstrate benefits of using the proposed distance measure over the existing ones in IVIF TOPSIS method for MCGDM problems.

1. Introduction

Multi-criteria decision making (MCDM), one of the well-known branches of decision-making, aims at finding the most suitable solutions from a set of alternatives under conflicting criteria. Due to the increasing complexity of the socio-economic environments, decisions are tend to be made by groups of decision-makers (DM) rather than individuals, hence, many real world decision problems are considered as multi-criteria group decision making (MCGDM) problems [1,2]. MCGDM process takes place in an environment where much of human knowledge is imprecise, and therefore, DMs may provide imprecise information collected from a variety of sources such as unquantifiable information about alternatives with respect to criteria. Therefore, fuzzy set theory proposed by Zadeh [3] has received a great attention in MCGDM literature as the fuzzy sets are more capable of reflecting human thinking about the crisp numbers [4,5].

Over the last decades, several higher-order fuzzy sets have been introduced in the literature. Intuitionistic fuzzy set (IFS) introduced by Atanassov [6] is one of the higher-order fuzzy sets used to deal with vagueness. IFS is characterized by three parameters; namely, membership function, non-membership function and hesitation margins, while fuzzy set is characterized by only membership function. This concept is a more effective in dealing with the vagueness of decisions, which may not be accurately expressed by DMs in assessing the alternative solutions. These can be caused by the following facts: (1) the DMs may not have precise or sufficient information about the problem; (2) DMs may be unable to discriminate explicitly the superiority of an alternative to others [7], in such cases that DMs may provide their assessments on alternatives to a certain extend, rather than complete certainty [8]. Due to this fact, DMs would find handier to express their assessments using IFSs rather than exact numerical values or linguistic variables [9–13], especially, interval valued intuitionistic fuzzy sets (IVIFSs). In fact, expressing opinions with the membership degree and non-membership degree enables DMs to easily and accurately reflect their assessments about decision problems. Based on this phenomenon, Atanassov and Gargov [14] extended IFSs to IVIFSs. In literature, a substantial the research has been conducted on the IVIFSs focusing mainly on the basic theory of IVIFSs such as the relations and operations of IVIFSs [15], the correlation and correlation coefficients of IVIFSs [16,17], the topology of IVIFSs [18], the relationships among the IVIFS, L-fuzzy set, IFS, interval-valued fuzzy set [19] and pattern recognition [20]. Recently, few studies on IVIFSs have been reported in decision-making literature. Xu [21] proposed a number of aggregation operators for interval valued intuitionistic fuzzy information applied to MCDM problems while, in [22], he investigated MCDM problems with which the information about criteria weights is incomplete, and the criteria values were expressed in interval valued intuitionistic fuzzy numbers (IVIFNs). Xu and Yager [23] introduced dynamic intuitionistic fuzzy aggregation operators and demonstrated how IVIFNs are applied to decision making problem while Xu and Chen [24] developed the ordered weighted aggregation operator and hybrid aggregation operator for aggregating preferences of DMs which were expressed as interval valued intuitionistic fuzzy information. Ye [25] proposed a novel accuracy function, which takes hesitancy degree of IVIFSs into the account for ranking IVIFNs. Wang et al. [26] proposed an approach to MCDM with incomplete criteria weight information where individual assessments are provided as IVIFNs. This proposed approach uses linear programming to determine criteria weights by applying a number of optimization models. Chen et al. [27] developed an approach for handling MCGDM problems in IVIFSs. Based on the operational rules of IVIFNs, several different aggregation operators were exercised with IVIFSs and compared according to the final ranks of alternatives. Ye [28] proposed a method for solving MCGDM problems using

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IVIF environments when expert opinions and criteria weights are unknown. Entropy weight models were used to determine the weights of both expert opinions and attributes and then the weighted correlation coefficients between each individual alternative and the ideal one were introduced for ordering the alternatives. Ye [29] constructed two weight models based on the score function to determine the weights of both experts and criteria. Then, the overall evaluation formulae of weighted scores for each alternative solution was introduced. Chen [30] introduced a new TOPSIS model relating an inclusion comparison approach for solving MCGDM problems using IVIF approaches. Wei and Zhang [31] proposed an entropy measure for solving MCGDM problems with IVIF approaches. Joshi and Kumar [32] extended Choquet integral operator, which lets model inter-dependency among the decision criteria, to interval-valued intuitionistic hesitant fuzzy sets for handling MCGDM problems. Moreover, multiplicative preference relation of IFS and IVIF sets for handling MCGDM problems has also been proposed by [33–38].

This study proposes a generalized distance measure for IVIF sets to close the deficiencies of well-known distance measures as discussed and detailed in Section 4. A TOPSIS method, in which the proposed distance measure is used to calculate separation measures, for solving group decision making problems with incomplete criteria weights is extended to IVIF environments. In order to accomplish this, firstly, a linear programming model to determine the optimal criteria weights is constructed based on the concept of maximizing relative closeness coefficient. Then, the extension of TOPSIS method to IVIF environments is demonstrated to solve MCGDM problems using optimal criteria weights derived with linear programming model.

The rest of this paper is organized as follows. Section 2 briefly describes IVIFSs and related definitions including aggregation operators, score and accuracy functions are given. Section 3 introduces a generalized distance measure for IVIFSs along with proofs illustrating the satisfaction of the axioms. A comparative analysis between the proposed distance measure and the existing ones is presented in Section 4 while Section 5 presents the extension of TOPSIS method to IVIF environments with incomplete criteria weights for group decision-making problems. Section 6 contains two illustrative examples to demonstrate the success of the proposed distance measure used with IVIF TOPSIS method for solving MCGDM problems in comparison with existing well-known distance measures. Finally, Section 7 provides conclusions derived of the study.

2. Preliminaries

In the following, some basic concepts on IFS and IVIFS are introduced.

Definition 1. An IFS A in a finite set X can be written as:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\} \tag{1}$$

where $\mu_A(x), \nu_A(x) : X \to [0, 1]$ are membership and non-membership degrees respectively, such that

$$0 \le \mu_A(x) + \nu_A(x) \le 1 \tag{2}$$

A third parameter of IFS is $\pi_A(x)$, known as the intuitionistic fuzzy index or hesitation degree of whether x belongs to A or not [6].

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \tag{3}$$

It is obviously seen that for every $x \in X$:

$$0 \le \pi_A(x) \le 1 \tag{4}$$

If the $\pi_A(x)$ is little, knowledge about x is more certain. If $\pi_A(x)$ is great, knowledge about x is more uncertain. Obviously, when $\mu_A(x) = 1 - \nu_A(x)$ for all elements of the universe, the traditional fuzzy set concept is recovered [39–43].

Definition 2. Let D [0, 1] be the set of all closed subintervals of the interval [0, 1], and X be ordinary finite non-empty sets. An IVIFS A in X is an expression given by:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\} \tag{5}$$

where $\mu_A(x), \nu_A(x) : X \to [0, 1]$ are membership and non-membership degrees, respectively.

$$0 \le \sup(\mu_A(x)) + \sup(\nu_A(x)) \le 1 \tag{6}$$

For each $x \in X$ are $\mu_A(x)$ and $\nu_A(x)$ closed intervals lower and upper end points of which are, denoted by $\mu_A^L(x), \mu_A^U(x), \nu_A^L(x), \nu_A^U(x)$, respectively.

As for each IVIFS in X, the following function expresses a hesitation degree of whether x belongs to A or not.

$$\pi_{A,\inf}(x) = 1 - \sup\ \mu(x) - \sup \nu(x) \tag{7}$$

$$\pi_{A,\sup}(x) = 1 - \inf\ \mu(x) - \inf \nu(x) \tag{8}$$

Here $\pi_A^L(x) = \pi_{A,\inf}(x)$ and $\pi_A^U(x) = \pi_{A,\sup}(x)$, where $(\pi_A^L(x), \pi_A^U(x)) \in D[0, 1]$ [14].

Definition 3. Let $\tilde{a}_j = ([\mu_j^L, \mu_j^U], [\nu_j^L, \nu_j^U])(j = 1, 2, 3, \ldots, n)$ be collection of IVIFNs [21]:

Let $\text{IIFWA} : \Omega^n \to \Omega$ if

$$\text{IIFWA}_w(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \ldots, \tilde{a}_n) = \sum_{j=1}^{n} w_j \tilde{a}_j = \left[\left(1 - \prod_{j=1}^{n}(1-\mu_j^L)^{w_j}, 1 - \prod_{j=1}^{n}(1-\mu_j^U)^{w_j}\right), \left(\prod_{j=1}^{n}(\nu_j^L)^{w_j}, \prod_{j=1}^{n}(\nu_j^U)^{w_j}\right)\right] \tag{9}$$

where $w = (w_1, w_2, w_3, \ldots, w_n)^T$ is the weight vector of $\tilde{a}_j (j = 1, 2, 3, \ldots, n)$ with $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$.

Then IIFWA is called as interval valued intuitionistic fuzzy averaging operator (IIFWA). In the case of equal weights, $w = (1/n, 1/n, 1/n, \ldots, 1/n)^T$ the IIFWA operator is reduced to the interval-valued intuitionistic fuzzy averaging (IIFA) operator, which is defined as follows:

$$\text{IIFA}(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \ldots, \tilde{a}_n) = \frac{1}{n}\sum_{j=1}^{n} \tilde{a}_j \tag{10}$$