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A novel approach to guarantee good robustness of fuzzy reasoning

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ABSTRACT

The analysis of internal connective operators of fuzzy reasoning is very significant and the robustness of fuzzy reasoning has been calling for study. An interesting and important question is that, how to choose suitable internal connective operators to guarantee good robustness of rule-based fuzzy reasoning? This paper is intended to answer it. In this paper, Lipschitz aggregation property and copula characteristic of t-norms and implications are discussed. The robustness of rule-based fuzzy reasoning is investigated and the relationships among input perturbation, rule perturbation and output perturbation are presented. The suitable t-norm and implication can be chosen to satisfy the need of robustness of fuzzy reasoning. In 1-Lipschitz operators, if both t-norm and implication are copulas, the rule-based fuzzy reasoning is much more stable and more reliable. In copulas, if both t-norm and implication are 1-l_∞-Lipschitz, they can guarantee good robustness of fuzzy reasoning. The experiments not only illustrate the ideas proposed in the paper but also can be regarded as applications of soft computing. The approach in the paper also provides guidance for choosing suitable fuzzy connective operators and decision making application in rule-based fuzzy reasoning.

1. Introduction

Fuzzy reasoning and applications have been actively pursuing since Zadeh's pioneering work [33–36]. The most fundamental inference forms of fuzzy reasoning are fuzzy modus ponens and fuzzy modus tollens based on rule. Up to now, numerous algorithms of rule-based fuzzy reasoning have been introduced in many literatures [8,15,18,20,28,37,38]. Among these methods, the choice of internal connective operators greatly affects the results of ruled-based fuzzy reasoning. The analysis of internal connective operators is a very active area of fuzzy reasoning and the robustness of fuzzy reasoning has been researching in many monographs. The main internal connective operators of rule-based fuzzy reasoning are triangular norm (briefly t-norm) and implication. The properties and the applications of t-norms and implications have been investigating in many literatures, for example, see [1,5,6,27,10–14,18,20–22,24–27,29–31].

In rule-based fuzzy reasoning, using different t-norms and implications can devote to drastic changes on the output values because of input perturbations and rule perturbations. These inspire us to think, what class of t-norms and implications can be used in rule-based fuzzy reasoning? Since choosing different t-norms and implications leads to different outputs, how to choose suitable t-norms and implications for rule-based fuzzy reasoning? What kind of internal connective operators can guarantee good robustness of fuzzy reasoning?

k-Lipschitz t-norms are very significant for fuzzy reasoning. The Lipschitz condition is to reveal the changes between the function value and the independent variables. It is stronger than continuity but weak than derivability and can guarantee iterative processes or equivalently, which make it reasonable to model dynamic process. On the other hand, the Lipschitz property of aggregation functions guarantees their stabilities with respect to small perturbation of inputs, which is very desirable for application [1]. k-Lipschitz t-norms and k-l_p-Lipschitz t-norms were well studied in monographs by Mesiarová [21,22]. In [21], some topological properties of k-Lipschitz triangular norms were investigated and the results for the case $k=1$ were generalized. In [22], k-l_p-Lipschitz t-norms were shown to be ordinal sums of k-l_p-Lipschitz Archimedean t-norms and additive generators of k-l_p-Lipschitz t-norms were characterized by means of k-p-convexity known for the case $k=1$. Beliakov

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et al. [1] considered the practical construction of k-Lipschitz triangular norms and conorms from empirical data. Ricci and Mesiar [27] discussed the necessary conditions for the Lipschitz of strict t-norms. In this paper, Lipschitz aggregation property and copula characteristic of t-norms are discussed in detail. In k-Lipschitz ($k \geq 1$) t-norms, some are 1-Lipschitz and h-l_p-Lipschitz ($h \geq 1$). In 1-Lipsschitz t-norms, some are 1-l_∞-Lipschitz and some are copulas. In copulas, some are 1-l_∞-Lipschitz.

As one of the very important internal connective operators, the studies on implications and their applications in fuzzy reasoning have been pursing. Ying [31] directly defined implications from negation, conjunction and disjunction and showed some strong implications in fuzzy logic. Yager [30] investigated the global requirement for implication operators in fuzzy modus ponens. In [29], Yager suggested two approaches for obtaining multivalued implications based on the direct use of additive generating functions, derived some new classes of implication operators and described the role of multivalued implication with the fuzzy logic based theory of approximate reasoning. Recently, Pei proposed unified implication algorithms of fuzzy reasoning in [26] and studied the formalization of implication based fuzzy reasoning method in [25]. Liu [19] introduced h^{-1}-implications. Tang introduced symmetric implicational method of fuzzy reasoning in [28]. A new full implication algorithm based on interval-valued fuzzy inference was presented in [20]. In this paper, the implications are investigated deeply from the perspective of Lipschitz aggregation property and copula characteristic. Some k-Lipschitz ($k \geq 1$) implications are found. In k-Lipschitz ($k \geq 1$) implications, some are 1-Lipschitz and h-l_p-Lipschitz ($h \geq 1$). In 1-Lipsschitz implications, some are 1-l_∞-Lipschitz and some are copulas. In copulas, some are 1-l_∞-Lipschitz.

Perturbation is the meaning of uncertain, imprecise and slight error. When a control system is sensitive to small deviations around input and rule, it is essential to find the maximum tolerance of the system with respect to those perturbations, referred as the system's robustness [15]. The researches on robustness of fuzzy reasoning have been shown in many literatures, for instance, see [2–4,6,7,9,15–17,32]. Cai [2] presented robustness results for various implication operators and inference rules in terms of δ-equalities of fuzzy sets. Cheng and Fu [4] estimated the upper and lower bounds of the output error affected by the perturbation parameters of input and obtained the limits of the output values when the input values ranged over some interval in many fuzzy reasoning schemes under CRI. Jin and Li et al. [9] proposed the definition of perturbation of fuzzy sets based on some logic-oriented equivalence measure and presented robustness results for various fuzzy reasoning machines by logically equivalence measure. Li et al. [15] discussed the robustness of internal-valued fuzzy connectives and investigated the sensitivity of interval-valued fuzzy sets. Dai and Pei et al. [7] extended the concept of perturbation of fuzzy sets according to normalized Minkowski distance to calculate the perturbation of fuzzy reasoning. In [6], Dai and Pei et al. studied the robustness of full implication inference method and fully implicational restriction method for fuzzy reasoning by means of fuzzy modus ponens and fuzzy modus tollens. Recently, Li and Qin et al. [16] presented robustness results for various fuzzy connectives and investigated the robustness of fuzzy reasoning from the perspective of perturbation of membership functions. The structures of four specific logic metric spaces induced by four important logic implication operators were discussed and their robustness were analyzed in [5]. In this paper, the robustness of rule-based fuzzy reasoning with internal connective operators which have Lipschitz aggregation property and copula characteristic is studied. The relationships among input perturbation, rule perturbation and output perturbation are revealed. According to Lipschitz aggregation property and copula characteristic, the suitable t-norms and implications can be selected to satisfy the need of robustness of fuzzy reasoning. Moreover, if both t-norm and implication are copulas and 1-l_∞-Lipschitz, they can guarantee good robustness of rule-based fuzzy reasoning. All these are illustrated by the experiments and these experiments are the applications of intelligent information processing, especially face association. The approach proposed in the paper also provides guidance for choosing suitable fuzzy connective operators and decision making application in rule-based fuzzy reasoning.

The correspondence is structured into several sections. In the next section, some definitions and lemmas are described. In Section 3, Lipschitz aggregation property and copula characteristic of t-norms and implications are concentrated on. In Section 4, the robustness of fuzzy reasoning with Lipschitz aggregation operators are studied. In Section 5, the experiments with the specific application of fuzzy reasoning are given and the theoretical analysis results are verified. The conclusions are presented in Section 6.

2. Some definitions and lemmas

Throughout our study, the following fuzzy operations are used on the unit interval [0,1]:

$a \wedge b = \min(a, b)$, $a \vee b = \max(a, b)$, $\sup_{i \in I} a_i = \vee_{i \in I} a_i$, $\inf_{i \in I} a_i = \wedge_{i \in I} a_i$, I is nonempty finite index set.

In this section, some definitions are recall and some new definitions and lemmas are presented.

Definition 2.1. [11]A binary operation T on the unit interval [0,1] is a t-norm, for all $x, y, z \in [0, 1]$, if it satisfies the following properties:

1) commutative: $T(x, y) = T(y, x)$,
2) associative: $T(T(x, y), z) = T(x, T(y, z))$,
3) monotone: $T(x, z) \leq T(y, z)$ whenever $x \leq y$,
4) neutral element: $T(x, 1) = x$.

In the correspondence, we focus on the following four basic t-norms[11]:

1) minimum or Gödel: $T_M(x, y) = T_G(x, y) = min(x, y)$,
2) product: $T_P(x, y) = xy$,
3) Lukasiewicz: $T_L = max(x + y - 1, 0)$,
4) drastic product: $T_D(x, y) = \begin{cases} \min(x, y), & if \quad \max(x, y) = 1, \\ 0 & otherwise. \end{cases}$

The dual t-norms are triangular conorms (t-conorms for short).

Definition 2.2. [11]A binary operation S on the unit interval [0, 1] is a t-conorm, for all $x, y, z \in [0, 1]$, if it is commutative, associative, monotone and satisfies $S(x, 0) = x$ for all $x \in [0, 1]$.

Definition 2.3. [6,19]A mapping R: $[0, 1]^2 \to [0, 1]$ is a fuzzy implication if it is non-increasing on the first variable and non-decreasing on the second one and it satisfies $R(0, y) = 1$, $R(x, 1) = 1$ and $R(1, 0) = 0$ for all $x, y \in [0, 1]$.

Definition 2.4. [19]A fuzzy implication R is said to have

1) the left neutrality property, if $R(1, y) = y$ for all $y \in [0, 1]$,
2) the exchange principle, if $R(x, R(y, z)) = R(y, R(x, z))$ for all $x, y, z \in [0, 1]$,
3) the identity principle, if $R(x, x) = 1$ for all $x \in [0, 1]$,
4) the order property, if $R(x, y) = 1 \Leftrightarrow x \leq y$ for all $x, y \in [0, 1]$,
5) the consequent boundary, if $R(x, y) \geq y$ for all $x, y \in [0, 1]$.

The most important classes of implications are R-, S- and QL-implications. In the correspondence, the following implications are concentrated on, most of them can be seen in [4]: