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Consistency-based linear programming models for generating the priority vector from interval fuzzy preference relations

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ABSTRACT

Interval fuzzy preference relations that can well cope with the vagueness and uncertainty are commonly used by the decision maker. The most crucial issue is how to derive the interval priority vector from an interval fuzzy preference relation. This paper first analyzes the size of the interval priority weights. Then, two linear programming models are built, by which the interval priority weights are obtained, respectively. Considering the inconsistent case, two consistency-based linear programming models are built to derive the additive consistent fuzzy preference relations. Different to the current methods, new models consider the consistency and the interval priority weight simultaneously. In some situations, the decision maker may only offer an incomplete interval fuzzy preference relation, namely, some judgments are missing. To cope with this situation, we first classify the missing intervals into three categories and then apply the associated linear equations to denote the missing values. After that, we construct two consistency-based linear programming models to determine the missing values to cope with the consistent and inconsistent cases. It is worth noting that the built models can cope with the situation where ignorance objects exist. Meanwhile, the associated numerical examples are offered, and the analysis comparison is made.

1. Introduction

Preference relations are an important and relatively simple decision-making method, which only need the decision maker to compare a pair of objects at a time. Then, the objects are ranked according to the priority vector that is generated from the preference relation. Generally speaking, there are three kinds of preference relations [1]: multiplicative preference relations [2,3], fuzzy preference relations [4–6] and linguistic preference relations [7–9]. With respect to these types of preference relations, there are many approaches to derive the priority vector [10–22]. All of the above preference relations need the decision maker to offer the exact judgments. However, the practical decision-making problems are becoming more and more complex. It is insufficient to fully express the decision maker's judgments by using exact values. To address this issue, Saaty and Vargas [23] introduced interval reciprocal (or multiplicative) preference relations, which permit the decision maker to use the intervals in [1/9, 9] to express his/her preferences. Then, the authors developed a Monte Carlo simulation approach to obtain the interval priority vector. After that, a number of methods to derive the interval priority vector are developed [24–34].

In addition to interval reciprocal preference relations, interval fuzzy (or complementary) preference relations [35] are another important class of preference relations, which can be regarded as an extension of fuzzy preference relations [5]. Xu [35] introduced a priority method to determine the priority vector from an interval fuzzy preference relation. After the pioneer work of Xu [35], there are many studies about interval fuzzy preference relations [36–39]. However, all these researches are insufficient to cope with the inconsistent case, and in sometimes, it is necessary to adjust the decision maker's judgments [41]. Later, Wang and Chen [40] introduced the concept of geometric transitivity of interval fuzzy preference relations, which is equivalent to multiplicative consistent interval fuzzy preference relations [39]. After that, the authors built the logarithmic least squares model to derive the interval priority vector. However, the method in [40] ignores the complementary of the consistency of the lower and upper bounds of the interval preferences. Using the convex combination method, Liu et al. [41] considered an interval fuzzy preference relation to be consistent when two related fuzzy preference relations are consistent. This method is based on the assumption that the decision maker has the same risk-attitude for all his/her interval judgments. Recently, Zhang et al. [42] applied the convex combination method to develop an approach to generate the interval priority vector from interval fuzzy preference relations. The main advantage of this method is that it endows the decision maker with different risk-preferences. However, when the methods in [36,41,42] adjust the inconsistent interval fuzzy preference relations, none of them consider the interval priority vector.

Based on the above analysis, this paper continues to study interval fuzzy preference relations, and develops a new method, which can overcome the issues in [36,41,42]. To do this, we first analyze the size of the interval priority weights. Then, several consistency-based liner programming models to calculate the interval priority weight are built.

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As researchers [43–56] noted, because of the complexity and uncertainty of decision-making problems and the lack of the associated expertise, the decision maker may only provide partial judgments for the compared objects, which is the so-called incomplete preference relations [52]. To cope with incomplete interval fuzzy preference relations, we further construct four linear programming models to estimate the missing values with respect to each object, and the missing values are endowed with the arithmetic mean of all obtained optimal solutions. Then, the interval priority weights are derived by using the previous built models. It is worth noting that no matter which kind of interval fuzzy preference relations is considered: consistent or inconsistent, complete or incomplete, the interval priority weights are all derived from additive consistent fuzzy preference relations.

The rest part of this paper is organized as follows: In Section 2, we first review some basic concepts about fuzzy preference relations and interval fuzzy preference relations. Then, we briefly analyze several studies. In Section 3, we mainly consider complete interval fuzzy preference relations. To derive the interval priority weights, four consistency-based linear programming models are constructed, by which the interval priority weight is obtained from the associated additive consistent fuzzy preference relations. In Section 4, we mainly discuss incomplete interval fuzzy preference relations. To obtain the missing values, linear programming models that are based on the consistency and the interval priority weight are constructed. Then, we assign the arithmetic mean of all obtained optimal solutions to denote the missing values. Meanwhile, numerical examples are given to show the concrete application of the built models, and the comparison with several methods is offered. The conclusion is made in the last section.

2. Basic concepts

Let $X = \{x_1, x_2, \ldots, x_n\}$ be the object set. A fuzzy preference relation $R = (r_{ij})_{n\times n}$ [5] on X is a fuzzy set on the product set $X \times X$, i.e., it is characterized by a membership function μ_R: $X \times X \to [0,1]$, where $r_{ij} = \mu_R(x_i, x_j)$ means the preferred degree of the object x_i over x_j. In this paper, we always assume that $R = (r_{ij})_{n\times n}$ satisfies the additive reciprocity property, namely, $r_{ij} + r_{ji} = 1$ for all $i, j = 1, 2, \ldots, n$. Considering the consistency of fuzzy preference relations, Tanino [57] introduced the following concept.

Definition 1 *([57])*. The fuzzy preference relation $R = (r_{ij})_{n\times n}$ is called additive consistency, if it satisfies

$$r_{ij} = r_{ik} + r_{kj} - 0.5 \tag{1}$$

for all $i, k, j = 1, 2, \ldots, n$ with $i < k < j$ and $r_{ij} + r_{ji} = 1$.

Let $w = (w_1, w_2, \ldots, w_n)$ be a vector such that $\sum_{i=1}^{n} w_i = 1$ and $w_i \geq 0$ for all $i = 1, 2, \ldots, n$. For the fuzzy preference relation $R = (r_{ij})_{n\times n}$, if we have

$$r_{ij} = 0.5(w_i - w_j + 1), \quad i, j = 1, 2 \ldots, n \text{ such that } i < j, \tag{2}$$

then $R = (r_{ij})_{n\times n}$ is additive consistency [58].

However, we cannot guarantee that Eq. (2) holds for an additive consistent fuzzy preference relation. For example, let the fuzzy preference relation R be given as follows:

$$R = \begin{pmatrix} 0.5 & 0.8 & 1 \\ 0.2 & 0.5 & 0.7 \\ 0 & 0.3 & 0.5 \end{pmatrix},$$

One can easily check that R is additive consistency. However, according to Eq. (2), we obtain $w_1 = 2.6/3$, $w_2 = 0.8/3$ and $w_3 = -0.4/3$.

Let $R = (r_{ij})_{n\times n}$ be additive consistency, then we can prove that the priority vector $w = (w_1, w_2, \ldots, w_n)$ of $R = (r_{ij})_{n\times n}$ can be expressed by

$$w_i = \frac{\sum_{k=1}^{n} 2r_{ik} + 1 - n}{n}, \quad i = 1, 2, \ldots, n. \tag{3}$$

To avoid the situation where $w_i < 0$ for some $i = 1, 2, \ldots, n$, Meng and Chen [55] defined the priority vector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ as follows:

$$\omega_i = \frac{w_i}{\sum_{j=1}^{n} w_j}, \quad i = 1, 2, \ldots, n. \tag{4}$$

where $w_i = \max\left\{0, \frac{\sum_{k=1}^{n} 2r_{ik}+1-n}{n}\right\}$ for all $i = 1, 2, \ldots n$.

Although researchers argued that there are many reasons to apply fuzzy preference relations in the AHP, it requires the decision maker to offer exact preferences by using the values in [0,1]. However, in practical decision-making problems, there usually exist various kinds of uncertain and fuzzy factors. To cope with this situation, interval fuzzy preference relations may be a good choice.

Definition 2 *([35])*. An interval fuzzy preference relation $A = (\bar{a}_{ij})_{n\times n} = ([l_{ij}, u_{ij}])_{n\times n}$ on X is an interval fuzzy set on the product set $X \times X$, i.e., it is characterized by an interval membership function $\bar{\mu}_R : X \times X \to [0, 1]$, where $\bar{r}_{ij} = \bar{\mu}_R(x_i, x_j)$ means the interval preferred degree of the object x_i over x_j. In general, it satisfies $l_{ij} + u_{ji} = 1$ and $l_{ii} = u_{ii} = 0.5$ for all $i, j = 1, 2, \ldots, n$.

With respect to interval fuzzy preference relations, there are two additive consistency concepts.

Definition 3 *([36])*. Let $A = (\bar{a}_{ij})_{n\times n} = ([l_{ij}, u_{ij}])_{n\times n}$ be an interval fuzzy preference relation, if there is a vector $w = (w_1, w_2, \ldots, w_n)$ such that

$$l_{ij} \leq 0.5(w_i - w_j + 1) \leq u_{ij}, \quad i, j = 1, 2, \ldots, n \text{ with } i < j$$

where w satisfies $\sum_{i=1}^{n} w_i = 1$ and $w_i \geq 0$ for all $i = 1, 2, \ldots, n$. Then, A is said to be additive consistency.