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Quasi-synchronization of heterogeneous complex networks with switching sequentially disconnected topology

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ABSTRACT

In this paper, quasi-synchronization is investigated for heterogeneous complex networks. All nodes of complex networks are nonlinear and have different dynamics. When topology is switching and disconnected, some sufficient criteria that guaranteeing the quasi-synchronization of heterogeneous complex networks are obtained based on the theory of sequential connectivity and joint connectivity. Even if the complex networks are heterogeneous, without any external control, and the topology is not connected at any moment, the errors between any pair of nodes will not exceed the upper bound, which is derived by calculating the maximal distance between any pair of nodes. Finally, some numerical simulations are provided to demonstrate the efficiency of the theoretical results.

1. Introduction

Complex networks is quite normal in our daily life, such as the communication network which represents the critical infrastructure networks, a variety of biological networks, and various reality and online social networks [1]. By applying the graph theory [2–4], a complex network is usually described by a graph that composed of a large number of nodes coupled by edges, which represent the elements and connections among nodes, respectively. But it should be pointed out that a dynamic complex network is different from the traditional graph theory, since each node of a dynamic complex network has its' own dynamic behaviors. Moreover, these dynamic behaviors can bring about different collective behaviors according to different topology structures. Generally, complex networks may converge to an equilibrium point, a periodic orbit, or a chaotic trajectory. Furthermore, as the proposition of small-world theory [5] and scale-free theory [6], more and more people from science and technology communities pay high attentions to the study of complex networks. And most of these studies focus on the relationships between the synchronization and the topology structure of complex networks [7–14].

As one of the classical phenomenon of complex networks, synchronization has been extensively studied due to its widely application among engineering fields [15–19], such as telephone network [18], consensus of mobile autonomous agents [19], secure communication, etc. However, most studies on synchronization are based on complex networks with identical nodes [8,13,20,21], i.e., homogeneous complex networks. By analyzing the structure topology [22,23] and the delayed complex networks [24], a variety of results have been presented. In fact, complex networks with nonidentical nodes are ubiquitous in real situations, such as the Internet, economic markets, and neural networks. Such heterogeneous complex networks are difficult to reach synchronization. On one hand, differential equations of heterogeneous complex networks do not exist a common equilibrium or a common solution. On the other hand, the theoretical analysis of heterogeneous complex networks is difficult. Consequently, some control strategies and concepts are proposed. In [25,26], quasi-synchronization, which means that heterogeneous complex networks synchronize to a virtual target with a bounded error, is introduced for heterogeneous complex networks. Some control strategies [25–28] are adopted to derive synchronization of heterogeneous complex networks. However, it should be singled out that most presented studies on synchronization or quasi-synchronization mechanisms of complex networks are mainly concentrate on fixed topology, which are heavily based on Lyapunov stability theory. For different coupling structures, complex networks are often presented by dynamic or switching topology. Recently, some findings in multi-agent system field show that multi-agent systems can achieve consensus [29–31]. Through the concept of relative hull, [31] provided a novel consensus algorithm for multi-agent systems. Besides, some results related to the consensus of multi-agent systems have been proposed under the condition of sequentially connected topology [2] and jointly connected topology [29]. For instance, by using spanning tree theory and sequential connectivity theory, [2] obtained a result

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that an unsteady distributed consensus system would converge in a more tighter way.

Since investigations on synchronization of complex networks with switching topology are not quite ample, particularly on switching disconnected topology and most studies on disconnected topology of complex networks required identical nodes. [11] showed that a complex network can reach synchronization even if the network is switching disconnected at any arbitrary time instant by adopting the concept of sequential connectivity and joint connectivity. Furthermore, [8] derived several sufficient conditions by estimating the maximal error bound between different pair of nodes. Moreover, traditional Lyapunov stability theory [10,11,20,23,25,26] is not suitable for theoretical analysis on complex networks with switching disconnected topology. Besides, most studies on quasi-synchronization are dependent on external control. Thus, it becomes even harder for nonlinear heterogeneous networks with switching disconnected topology. Inspired by all discussions above, theoretical analysis on synchronization of heterogeneous networks with switching disconnected topology is necessary.

This paper investigates the quasi-synchronization of heterogeneous complex networks with switching sequentially disconnected topology. The main contributions of this paper are as follows: By applying the theory of sequential connectivity and joint connectivity, some sufficient criteria that guaranteeing the quasi-synchronization of heterogeneous complex networks are obtained. Both the connectivity of heterogeneous complex networks and control strategy are not required in the paper. Besides, the errors between any pair of nodes will not exceed the upper bound and the upper bound is obtained via calculating the maximal distance between any pair of nodes.

The remainder of this paper is organized as follows: Section 2 introduces the mathematical model and some preliminaries; Section 3 derives the main results; Section 4 provides a simulation to verify the effectiveness of theoretical results; Section 5 concludes this paper.

2. Preliminaries

2.1. Graph theory

To solve the quasi-synchronization problems, this paper introduces the related graph theory (see Angeli & Bliman [2] for details). Given a graph $G=(V,E)$ in which $V=\{1,2,\dots,N\}$ and $E\subseteq V\times V$ represents the set of nodes and edges, respectively. Besides, G is connected if for any node $i,j\in V$, there is a path between node i and node j despite its directions. And G contains a spanning tree if G is connected. Denote $g_i=\{V,E_i\}$ is the ith sub-graph of G and $g_i\cap g_j=\{V,E_i\cap E_j\}$ represents the combination of g_i and g_j. $A=(a_{ij})_{i,j=1}^N\in R^{N\times N}$ is a nonnegative matrix that corresponds to the graph $G(A)=\{V,E\}$, i.e., $(i,j)\in E$ if and only if $a_{ij}>0$. This paper concentrate on complex networks with directed graphs, besides, self-loop is not considered in this paper.

Sequential connectivity (Angeli & Bliman [2]) A sequence of graphs $\{g_i\}_{i=1}^m$ is said to be sequentially connected if there existed a node $k\in V$ and the iteration given by

$$V_0=\{k\},$$
$$V_t\subseteq V_{t-1}\cup Neighbor(V_{t-1},G_t),\ t=1,2,\dots,T$$

with $G_T=G$. Besides, a sequence of connected graphs $\{g_i\}_{i=1}^m$ is also sequentially connected if $m\ge(N-1)^2$.

Joint connectivity (Jadbabai et al. [4]) A sequence of graphs $\{g_i\}_{i=1}^m$ is said to be jointly connected if the member of its union $\cup_{i=1}^m\{g_i\}$ is connected.

2.2. Model formulation

Consider the following complex network with N nonidentical nodes

$$\dot x_i(t)=f_i(x_i(t))+c\sum_{j=1}^N a_{ij}(t)(x_j(t)-x_i(t)),\quad i=\{1,2,\dots,N\},\tag{1}$$

where $x_i(t)\in R^n$ denotes the state of the node i, $f_i(x_i(t))$ is a nonlinear vector function of node i, $c\in R$ represents the coupling strength, and $a_{ij}(t)\in R$ are piecewise functions satisfies

$$a_{ij}(t)\equiv\begin{cases}1 & if\ \ (j,i)\in E\\ 0 & if\ \ (j,i)\notin E\end{cases},\ \text{for } i\ne j \text{ and } a_{ii}(t)=-\sum_{j=1,j\ne i}^{\mathrm{N}}a_{ij},$$

$\forall\ t_k\in[t_k,t_{k+1}),\ i\in V,$

where $t_k=(k-1)h$ and h is the sampling step. Denote that

$A_k=(a_{ij}(t))_{i,j=1}^N,\quad t\in[t_k,t_{k+1}).$

Quasi-synchronization of complex networks (1) means that there is a positive constant ϑ such that

$$\lim_{t\to\infty}\|x_i(t)-x_j(t)\|\le\vartheta,\ \forall\ i,j\in V.$$

Remark 1. Since the nonlinear dynamics f_i and f_j are different for $i\ne j$, the complex network (1) is heterogeneous. It is difficult to achieve synchronization for heterogeneous complex networks (1) because Eq. (1) does not exist a common solution or a common equilibrium.

2.3. Assumptions and Lemmas

Firstly, some assumptions are given to get the main results.

Assumption 1. Suppose that the nonlinear function $f_i(x)$ in system (1) satisfies Lipschitz condition. In other words, there exists a nonnegative constant l_i, such that for any $x,y\in R^n$,

$$\|f_i(x)-f_i(y)\|\le l_i\|x-y\|,\quad i\in\{1,2,\dots,N\}.$$

Denote $L=\max_{1\le i\le N}\{l_i\}$.

Assumption 2. There is a positive constant M such that for any i,j and x, the following inequality holds

$$\|f_i(x)-f_j(x)\|\le M.$$

Remark 2. This paper investigates the synchronization problem of heterogeneous complex networks. In order to achieve synchronization, f_i and f_j should not be much difference for $i\ne j$; otherwise, it is difficult to achieve synchronization.

Assumption 3. There exists an integer $T>0$, such that the sequence of graphs $\{g_k\}_{k=(r-1)T+1}^{rT}$ is sequentially connected for any $ch\le1$, $r\ge1$ and the following equations hold

$$L<2c(N-1)\text{ and }0<[e^{\widehat{L}Th}-(che^{\lambda h})^T]<1,$$

where $\widehat{L}=L+\frac{\varepsilon}{2}$ for $\varepsilon>0$.

Remark 3. Since complex networks have many nodes, it is nature to suppose that $2c(N-1)$ is bigger than L. In other words, this paper adopts $L-2c(N-1)<0$ in following derivations.

Lemma 1. (Chen et al. [8]) For $x(t)\colon R\to R^n$, it holds that

$$D^+(\|x(t)\|)\le\|D^+(x(t))\|,$$

where $D^+(x(t))=\lim_{h\to0^+}\sup\frac{x(t+h)-x(t)}{h}$.

Lemma 2. (Chen et al. [8]) Given $x,y\in R^n$, if $\|x\|\ge\|y\|$, then $x^T(y-x)\le0$.

Lemma 3. The solution for differential equation $\dot x=\alpha x+\beta e^{Lt}+\gamma e^{\alpha t}+\omega$ with initial state $x(t_0)=x_0$ is given by