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Brief papers

Neutral-type of delayed inertial neural networks and their stability analysis using the LMI Approach☆

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ABSTRACT

A theoretical investigation of neutral-type of delayed inertial neural networks using the Lyapunov stability theory and Linear Matrix Inequality (LMI) approach is presented. Based on a suitable variable transformation, an inertial neural network consisting of second-order differential equations can be converted into a first-order differential model. The sufficient conditions of the delayed inertial neural network are derived by constructing suitable Lyapunov functional candidates, introducing new free weighting matrices, and utilizing the Writinger integral inequality. Through the LMI solution, we analyse the global asymptotic stability condition of the resulting delayed inertial neural network. Simulation examples are presented to demonstrate the effectiveness of the derived analytical results.

1. Introduction

Since the revitalization of research on *neural networks* (NNs) in 1980s, numerous studies on dynamical analyses of various types of NNs such as recurrent NNs [1], cellular NNs [2], Cohen-Grossberg NNs [3], and bidirectional associative memory (BAM) NNs [4], have been reported. They have also been successfully applied to solving different problems including image processing and signal processing (for more details, see [5,6]). In most applications, stable NN dynamics are required to produce useful results. As an instance, in hardware implementation of Hopfield NNs [7], a continuous time dynamical neural unit is represented by an analog RC (resistance capacitance) network circuit with a finite switching speed of amplifiers. In such case, the existence of time delays is inevitable. Time delays lead to complex dynamical behaviours, which include instability, periodic solutions, chaos, and cause poor performance of the resulting NNs. Time delay analysis in Hopfield NNs was first introduced by Marcus and Westervelt [8]. In the existing literature, the following delayed NNs, which can be considered as an extension of the Marcus-Westervelt type of Hopfield NNs, have been widely studied,

$$\dot{m}_i(t) = -a_i m_i(t) + \sum_{j=1}^{n} w_{ij}^1 g_j(m_j(t)) + \sum_{j=1}^{n} w_{ij}^2 g_j(m_j(t-h)) + I_i \tag{1}$$

where $m_i(t)$ denotes the state variable of the ith neuron at time t, $a_i > 0$ is a constant; $g_j(m_j(t))$ represents the activation function of the neuron, which describes the reaction of the jth neuron to $m_i(t)$; w_{ij}^1 and w_{ij}^2 denote the connection weights of the neurons; $h > 0$ is a constant time delay, and I_i is the external input. Many investigations pertaining to the stability results of delayed NNs (1) based on the Lyapunov stability theory have been conducted. As an example, the authors in [9,10] investigated the existence and uniqueness of NNs with multiple time delays. Specifically, the global robust asymptotic stability condition for the equilibrium points has been analysed. A generalized activation function, which can be used to derive the global robust stability condition for delayed NNs with time delays, has been examined in [11]. Indeed, dynamical characteristics of NNs with time delays constitute a popular research topic, and many novel stability criteria have been proposed. From the existing literature, it is clear that the existence of different types of time delays affects the performance of NNs.

It is worth noting that time delays exist in the derivative of a state pertaining to a dynamical system, in addition to appearing in the state itself. This type of time delay is known as neutral delays, and their existence can be observed in many physical systems, which include chemical reactors, transmission lines, partial element equivalent circuits in VLSI (Very-Large-Scale Integration) systems, and Lotka-Volterra systems (see [12–15]). The authors in [14,15] addressed the existence of neutral delays in a partial element equivalent circuit. The

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circuit was modelled as a neutral-delay type of differential equations, and a novel stability criterion was derived based on the Lyapunov theory. The existence of neutral delays is unavoidable when we model NNs with differential equations for the purpose of realization in electronic circuits. In this regard, neutral-type of NNs represent a special class of time-delayed NNs. A neutral-type NN can be represented using the following equation:

$$\dot{m}_i(t) = -a_i m_i(t) + \sum_{j=1}^{n} w_{ij}^1 g_j(m_j(t)) + \sum_{j=1}^{n} w_{ij}^2 g_j(m_j(t-h)) + \sum_{j=1}^{n} d_{ij}\dot{m}_j(t-h) + I_i, \tag{2}$$

where d_{ij} denotes the delayed connection weight matrix of the neurons. For this type of NNs, the stability criteria are more difficult than those of the retarded type. Therefore, neutral-type of NNs have received much research attention in recent years, and many stability criteria have been proposed for both delay-independent and delay-dependent cases. The delay-dependent stability conditions for neutral-type of NNs based on the Lyapunov stability theory and linear matrix inequality (LMI) have been derived in [16–18]. Furthermore, a state estimator was developed in [19] and its stability issue for neutral-type of delayed NNs was examined. Note that from models (1) and (2) and from the existing literature, most of the investigations have been focused on the qualitative analysis of NNs with first-order differential equations was examined. Studies on NNs with the influence of inductance, or, equivalently, an inertial term, are limited. Wheeler and Schieve [20] added the inertial term into continuous-time Hopfield NNs, and explored the stability effects pertaining to the fixed points of Hopfield NNs. Nevertheless, complicated oscillatory behaviours and even unstable behaviours [20–24] of delayed inertial NNs have not been analysed comprehensively. Only a few investigations on dynamical characteristics of NNs with the inertial term have been reported so far. As an example, the global stability analysis with an exponential term for BAM NNs based on second-order differential equations was presented in [22]. The existence of periodic solutions for time-delayed inertial Cohen-Grossberg-type of BAM NNs and their exponential stability were discussed in [23]. The stability and synchronization criteria for inertial BAM NNs with time delays based on matrix measure strategies were examined in [24]. The pinning synchronization criteria for coupled delayed inertial NNs were proposed in [25]. The following inertial type of NNs with a time-varying delay has recently been considered in [24], with the stability solutions analysed:

$$\ddot{m}_i(t) = -a_i\dot{m}_i(t) - b_i m_i(t) + \sum_{j=1}^{n} w_{ij}^1 g_j(m_j(t)) + \sum_{j=1}^{n} w_{ij}^2 g_j(m_j(t-h(t)) + I_i, \tag{3}$$

where $b_i > 0$ is a constant; $h(t)$ denotes the time-varying delay. Stability and synchronization issues for the aforementioned model have been discussed in [24,26,27]. The authors in [24] derived several sufficient conditions to ensure the global exponential stability and synchronization for model (3) using the matrix measure approach and the Halanay inequality. Furthermore, the exponential stabilization issues of model (3) were analysed with periodically intermittent control in [26]. The Lagrange stability criteria for inertial NNs with time-varying delays were discussed in terms of LMIs and the Lyapunov stability theory in [27]. Based on above account, the qualitative dynamical behaviours, which include local stability, global stability, bifurcations and chaos for second-order differential equations have been analysed in recent years. In this regards, the neutral type of NNs is a special class of delayed NNs, and can be represented by the following second-order differential equation

$$\ddot{m}_i(t) = -a_i\dot{m}_i(t) - b_i m_i(t) + \sum_{j=1}^{n} w_{ij}^1 g_j(m_j(t)) + \sum_{j=1}^{n} w_{ij}^2 g_j m_j(t-h) + \sum_{j=1}^{n} d_{ij}\ddot{m}_j(t-h) + I_i. \tag{4}$$

It should be noted that investigations on neutral type of inertial NNs through the Lyapunov stability theory have more conceptual significance. Therefore, we focus on the problem of global asymptotically stability analysis for the neutral type of inertial NNs with time delays in this paper. We start with a suitable variable transformation to simplify the stability analysis of inertial NNs. Then, the sufficient conditions are derived to ensure the global exponential stability of delayed inertial NNs by constructing suitable Lyapunov Krasovskii (L-K) functionals with the derivative of double integral terms. The Writinger integral inequality is employed. The effectiveness of the derived analytical results is demonstrated using numerical examples.

The organization of this paper is as follows. The related notation and background information related to the delay-dependent stability conditions for neutral type of inertial NNs are presented in Section 2. In Section 3, numerical examples and the associated simulation results are provided to show the effectiveness of the analytical results. Finally, concluding remarks are presented in Section 4.

2. Preliminaries and delay-dependent stability conditions

In this paper, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and $\mathbb{R}^{n\times n}$ represents the set of all $n \times n$ real matrices. Superscript A^T refers the transposition of matrix A, and $X \geq Y$ $(X > Y)$ indicates positive semi-definite (positive definite) matrices, where X and Y are symmetric matrices. I_n indicates the $n \times n$ identity matrix; $0_{n\times m}$ indicates the $n \times m$ dimensional zero matrix, and $\text{diag}\{..\}$ denotes the diagonal matrix.

The initial values of the proposed model of (4) are considered as in the following form,

$$m_i(s) = \varphi_i(s), \quad \dot{u}_i(s) = \psi_i(s), \quad -h \leq s \leq 0 \tag{5}$$

where $\varphi_i(s)$ and $\psi_i(s)$ are some bounded and continuous functions, respectively. Throughout this paper, the following assumption holds:

Assumption 1. The activation function of neuron $g_i(\cdot)$ in (4) satisfies

$$l_i^- \leq \frac{g_i(\sigma_1) - g_i(\sigma_2)}{\sigma_1 - \sigma_2} \leq l_i^+, \quad \forall\ \sigma_1, \sigma_2 \in \mathbb{R},\ \sigma_1 \neq \sigma_2,\ i = 1, \ldots, n, \tag{6}$$

By using the variable transformation function of $p_i(t) = \dot{m}_i(t) + m_i(t), \quad i = 1, 2, \ldots, n$, (4) and (5) can be re-written as follows

$$\begin{cases} \dfrac{dm_i(t)}{dt} &= -m_i(t) + p_i(t), \\ \dfrac{dp_i(t)}{dt} &= -(a_i - 1)p_i(t) - [b_i + (1 - a_i)]m_i(t) + \sum_{j=1}^{n} w_{ij}^1 g_j(m_j(t)) \\ & \quad + \sum_{j=1}^{n} w_{ij}^2 g_j(m_j(t-h)) + \sum_{j=1}^{n} d_{ij}(p_j(t-h) - p_j(t-h) \\ & \quad + m_j(t-h)) + I_i, \end{cases} \tag{7}$$

with initial values

$$\begin{cases} m_i(s) &= \tilde{\varphi}_i^1(s), \\ p_i(s) &= \psi_i(s) + \varphi_i(s) \triangleq \overline{\varphi}_i^2(s), \end{cases} \tag{8}$$

for $-h \leq s \leq 0$.Model (7) can be expressed in the matrix-vector form