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Frank aggregation operators and their application to hesitant fuzzy multiple attribute decision making

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ABSTRACT

In this paper, we investigate multiple attribute decision making (MADM) problems based on Frank triangular norms, in which the attribute values assume the form of hesitant fuzzy information. Firstly, some basic concepts of hesitant fuzzy set (HFS) and the Frank triangle norms are introduced. We develop some hesitant fuzzy aggregation operators based on Frank operations, such as hesitant fuzzy Frank weighted average (HFFWA) operator, hesitant fuzzy Frank ordered weighted averaging (HFFOWA) operator, hesitant fuzzy Frank hybrid averaging (HFFHA) operator, hesitant fuzzy Frank weighted geometric (HFFWG) operator, hesitant fuzzy Frank ordered weighted geometric (HFFOWG) operator, and hesitant fuzzy Frank hybrid geometric (HFFHG) operator. Some essential properties together with their special cases are discussed in detail. Next, a procedure of multiple attribute decision making based on the HFFHWA (or HFFHWG) operator is presented under hesitant fuzzy environment. Finally, a practical example that concerns the human resource selection is provided to illustrate the decision steps of the proposed method. The result demonstrates the practicality and effectiveness of the new method. A comparative analysis is also presented.

1. Introduction

Multiple attribute decision making (MADM) problems are widespread in daily life decision situations. This process aims at selecting the best alternative(s) from a set of available alternatives with respect to multiple attributes, both qualitative and quantitative. Many MADM approaches are successfully used to web 2.0 [1], social network [2], and recommendation system [3,4]. In many real-world situations, due to the increasing complexity of the socio-economic environment, there are numerous limiting factors such as a lack of information and knowledge, uncertainty of the decision-making environment, difficulties in information extraction, etc. Therefore, it becomes difficult for decision maker to express values of attributes in a numeric fashion. Instead, it is more suitable to express preferences by fuzzy numbers or extended fuzzy numbers [5–8,14,16]. Many theoretical studies on MADM problems have been put forward in recent years. A variety of approaches about MADM under hesitant fuzzy environment have been developed [9–11,15,17,18]. Torra [12] proposed the concept of hesitant fuzzy set (HFS). HFS is an extension of fuzzy set which permits the membership grades assuming a set of possible values rather than a single one. As such HFS it can be considered as a powerful tool for handling fuzzy uncertain information. Torra and Narukawa [13] studied the relationship between the HFS and other extended fuzzy sets (FSs), the results showed that the envelope of HFS is an intuitionistic fuzzy set (IFS), all the HFSs are type-2 fuzzy set (T2FS). Xu and Xia [19] proposed a series of distance and similarity measures for HFS, and further developed the hesitant ordered weighted distance measures and the hesitant ordered weighted similarity measures. Peng et al. [20] developed the generalized hesitant fuzzy synergetic weighted distance (GHFSWD) measure, and discussed some desirable properties together with their potential applications. Farhadinia [21] investigated the relationship between entropy, similarity measure and distance measure for HFS and the interval-valued hesitant fuzzy set (IVHFS). Qian et al. [22] extended HFS by IFS and applied it in decision support system (DSS). Chen et al. [23] studied interval-valued hesitant preference relations and utilized it to group decision making

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(GDM) problems. Zhu et al. [24] proposed the concept of dual hesitant fuzzy set (DHFS) and further investigated some basic operations and established a number of useful properties. Rodríguez et al. [25] introduced the concept of hesitant fuzzy linguistic term set (HFLTS), which provides a linguistic the richness of linguistic elicitation based on the fuzzy linguistic approach. Zhang and Wei [26] developed the VIKOR method to solve the MADM problems with hesitant fuzzy information. Zhang and Xu [27,28] extended the TOPSIS and TODIM methods to accommodate hesitant fuzzy environment, and further produced two hesitant fuzzy decision making methods. Dong et al. [29] developed the consensus model with hesitant linguistic assessments in group decision making. Zhu and Xu [30] constructed some consistency measures for hesitant fuzzy linguistic preference relations. Wang and Xu [31] proposed the concept of extended hesitant fuzzy linguistic preference relations. In addition, the HFS and its various extended forms have been widely used in group decision making [32–35], preference relations [36], clustering analysis [37], and other applications [38–42]. Recently, Rodríguez et al. [43] presented an overview of HFS with the aim of providing a clear perspective on the different concepts, tools and trends related to this extension of FS. In addition, Xu [44] proposed a detailed survey to review on the recent development of aggregation techniques and applications for HFS, and further pointed out some potential trends in this field.

In addition, as an important technique for information fusion, researches on hesitant fuzzy aggregation operators have been receiving more attention. Xia and Xu [45] proposed a series of aggregation operators and discussed their desirable properties in detail. Xia et al. [46] applied quasi-arithmetic means to form several series aggregation operators under hesitant fuzzy environment. Wei [47] developed some prioritized operators with hesitant fuzzy information, and applied them to handle MADM problems where the attributes exhibit different levels of priority. Wei et al. [48] investigated some hesitant interval-valued aggregation operators and developed a procedure to solve interval-valued hesitant fuzzy MADM problems. Motivated by the idea of fuzzy integral theory, Wei et al. [49] used the Choquet integral to derive two aggregation operators within the context of hesitant fuzzy information: hesitant fuzzy Choquet ordered averaging (HFCOA) operator and hesitant fuzzy Choquet ordered geometric (HFCOG) operator. Those two aggregation operators are not only considering the importance of the attributes, but also can capture the correlations among the independent attributes. Zhu et al. [50] developed a hesitant fuzzy Bonferroni mean (BM) and utilized geometric Bonferroni mean (BM) to solve MADM problems. Zhang [51] presented a wide range of hesitant fuzzy power aggregation operators for aggregating hesitant fuzzy information, several interesting properties and the relationships between them were also discussed. Based on Archimedean t-conorms and t-norms, Zhang and Wu [52] developed two weighted hesitant fuzzy aggregation operators for aggregating weighted hesitant fuzzy information, some desirable properties and special cases of the proposed aggregation operators are studied in detail. Recently, Zhang et al. [53] proposed some induced generalized hesitant fuzzy operators and applied them to MADM problems. Liao and Xu [54] investigated some hesitant fuzzy hybrid weighted aggregation operators and establish a decision algorithm to aid MADM problems with hesitant fuzzy information. Zhou and Xu [55] proposed two new aggregation approaches: optimal discrete fitting aggregation operator and simplified optimal discrete fitting aggregation operator to resolve GDM problems. Wei et al. [56] defined two aggregation operators for hesitant fuzzy linguistic term set (HFLTS): the hesitant fuzzy LWA operator and the hesitant fuzzy LOWA operator, and applied them to solve MADM problems with different situations in which importance weights of criteria or experts are known or unknown. Yu [57] developed some hesitant fuzzy aggregation operators based on Einstein operational laws. Tan et al. [58] proposed some hesitant fuzzy Hamacher aggregation operators and applied them to MADM problems. Qin and Liu [59] developed some hesitant fuzzy operators based on Maclaurin symmetric mean and studied various desirable properties of them. Compared with hesitant fuzzy Bonferroni mean [60], the prominent feature of hesitant fuzzy MSM is that it can capture the overall interrelationships among the arguments, so it is more general and suitable for practical decision applications.

All aforementioned aggregation operators are mainly based on the algebraic operational laws of general t-norms and their dual t-conorms [61]. The most widely used t-norm and t-conorm is product and probabilistic sum [61], because this pair of triangular norm is useful to carry out computing. However, the main drawbacks of the probability triangular norms are that lack of flexibility and robustness. To overcome these drawbacks, some aggregation operators based on other t-norms and t-conorms have been developed over the last decades. Xia et al. [62] extended some families of aggregation operators based on Archimedean t-conorm and t-norm with intuitionistic fuzzy information. Wang and Liu [63] developed a family of intuitionistic fuzzy information aggregation operators with the help of Einstein t-conorm and t-norm. Liu [64] investigated some Hamacher aggregation operators and discussed some of its desirable properties in detail. Wei and Zhao [65] explored some hesitant interval-valued fuzzy Einstein aggregation operators, such as the hesitant interval-valued fuzzy Einstein weighted average (HIVFEWA) operator, hesitant interval-valued fuzzy Einstein weighted geometric (HIVFEWG) operator, and the induced hesitant interval-valued fuzzy Einstein ordered weighted geometric (I-HIVFEOWG) operator, for aggregating hesitant fuzzy information.

Frank t-norm and t-conorm [66], which are an interesting generalizations of probabilistic and Lukasiewicz t-norm and t-conorm, is a general and flexible family of continuous triangular norms. Since the Frank t-norm and t-conorm involve a certain parameter, this makes them more flexible in the process of information fusion and is more adequate to model practical decision making problems. Tomasa et al. [67] studied the functional equations of Frank and Alsina for two classes of commutative, associative and increasing binary operators. Yager [68] investigated the additive generating function (AGF) of Frank t-norms, and established a framework with Frank t-norms in approximate reasoning. Sarkoci [69] made a comparison between the Frank t-norms and the Hamacher t-norms, and proved that two different t-norms form the same family. Casasnovas and Torrens [70] presented an axiomatic approach to scalar cardinality of Frank t-norms and proved that the properties remain true for other standard t-norms. Deschrijver [71] introduced the extensions operations to the lattice of closed interval-valued fuzzy set (IVFS) based on Frank t-norms and provided necessary and sufficient conditions such that the these operations can formed a complete algebraic lattice structure. Some researchers also studied various mathematical properties of the Frank t-norms [72–76].

It is noted that the previous research works almost exclusively focus on the mathematical structure and the properties of the Frank t-norms, while application-oriented studies are quite rare, especially in the context of information fusion and decision-making. We have shown that the HFS can be suitable to express uncertain information, and the Frank aggregation operators themselves extend the probabilistic and Lukasiewicz t-norms with the aid of parameter flexibility. In this sense, it becomes meaningful to extend the Frank t-norms to cope with the hesitant fuzzy environment. To date, we have not seen any related studies of aggregation operators based on Frank t-norms in problems of hesitant fuzzy decision-making problems. Motivated by this fact, it becomes beneficial to study a family of aggregation operators based on Frank operations with regard to hesitant fuzzy information and subsequently apply them to multiple attribute decision making problems. Those are the two strong motivating factors behind the study reported in this paper.

There is also a certain point which needs to be highlighted. One might be under an impression that all hesitant fuzzy sets can be represented as type-2 fuzzy sets. However, in practical applications it is difficult to extend the type-2 fuzzy aggregation operator to