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Neural network based adaptive prescribed performance control for a class of switched nonlinear systems

Li Tang, Jun Zhao*

State Key Laboratory of Synthetical Automation for Process Industries (Northeastern University), College of Information Science and Engineering, Northeastern University, Shenyang 110819, PR China

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ABSTRACT

In this paper, an adaptive prescribed performance control problem is studied for a class of switched uncertain nonlinear systems under arbitrary switching signals. Radial basis function neural networks (RBFNNs) are employed to approximate the unknown terms. By introducing the transformation errors, the original system with prescribed performance constraints are transformed into a class of new systems without constraints. Then, we design controllers to stabilize the transformed system. In this way, the original system is stabilizable and the prescribed performance is guaranteed. Update laws are designed such that the parameter estimation is fixed until its corresponding subsystem is active. Compared with the existing results, the main contributions of this paper are characterized as follows: (1) a PPC approach is proposed for a class of switched uncertain nonlinear systems in a general form and (2) RBFNNs are used to approximate the unknown terms of each subsystem. In the end, a simulation example is developed to illustrate the effectiveness of the present approach.

1. Introduction

Adaptive tracking control problems for uncertain nonlinear systems have been widely studied in the past two decades [1–8]. It is worth noting that the traditional adaptive tracking control designs guarantee that the tracking error convergence to a residual set as time goes to infinity. Its focus is on the steady-state performance, the transient-state performance is ignored.

To guarantee the transient performance, Bechlioulis and Rovithakis [9] firstly proposed a performance transformation function to realize the prescribed performance control (PPC). Then, several methods for coping with a variety of PPC problems were developed [10–13]. The PPC strategy is to design a controller such that the tracking error converges to an arbitrarily small residual set and that the convergence rate is no less than a predefined value, as well as that the maximum overshoot and undershoot are less than some predefined constants. In addition, the PPC method has been applied to many practical systems, such as robot force/position control systems [14], active suspension systems [15] and so on. Nevertheless, these investigations were all concentrated on the single modal systems. Actually, most practical plants cannot be described as single modal systems. They are required to be directly expressed by multiple modal systems, also called switched systems.

Switched systems are dynamical systems consisting of a family of subsystems, either continuous-time or discrete-time subsystems, and a switching rule, which means that a specific subsystem is active at a certain instant. In the last two decades, the analysis and synthesis problems of switched systems have widely attracted more and more attention because of their important roles in the engineering applications, see for example [16–19] and the references therein. In recent years, the study for switched nonlinear systems [20–24] is more and more popular. Due to the complicated framework of the switched nonlinear systems and the interaction between the system structure and switching, the control for this class systems is more complex and difficult. In [25], an output tracking control method was proposed for a constrained switched nonlinear system based on barrier Lyapunov functions. Besides, the input-to-state stability problem was studied in [26] for a switched stochastic nonlinear system. The PPC problem has been widely studied for non-switched systems, it is also expected to study the PPC problem for switched systems.

In switched systems, though the PPC bounds are satisfied for each subsystem, for different choices of the switching signals, the switched systems may not satisfy the PPC bounds. In order to satisfy the PPC bounds regardless of the switching instants, we need to find a common Lyapunov function [27,28]. In many cases, it is easy to find a Lyapunov function for each subsystem [29]. However, a common Lyapunov is hard to find for switched systems. On the other hand, to improve the system identification accuracy for switched uncertain nonlinear sys-

* Corresponding author.
E-mail addresses: tangli413@hotmail.com (L. Tang), zhaojun@mail.neu.edu.cn (J. Zhao).

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tems, different NNs should be employed to approximate the unknown terms for different subsystems, this means that the NNs are also switched. This greatly increases the difficulty in the procedure of designing control. Naturally, the switched update laws should be designed. Therefore, two issues naturally arise: Under what conditions can we find a common Lyapunov function? How to design update laws for switched NNs?

Motivated by the above discussion, we propose an adaptive NNs-based PPC approach for a class of switched systems in this paper. A common Lyapunov function was found to ensure the system stability. Based on the universal approximation ability of NNs, the unknown terms are approximated by RBFNNs. In addition, both the steady-state performance and the transient performance of the tracking error are obtained. It is shown that all the signals in the resulting closed-loop system are bounded. The main contributions of this paper compared with the existing results on switched and non-switched nonlinear systems are in two aspects: (1) The PPC problem for a class of switched uncertain nonlinear systems is studied in a general form. (2) RBFNNs are used to approximate the unknown terms of each subsystem and the switched update laws are designed.

Notation: $\Re^n$ stands for the n-dimensional space, $\Re^q$ represents the q-dimensional space, $\Re^p$ refers to the p-dimensional space. $\|\cdot\|$ denotes the Euclidean vector norm. $\gamma(\cdot)$ is a function of class κ. Ω_X is an compact set. C^1 stands for a set of functions with continuous first order derivative. For a given matrix A (or vector v), A^T (or v^T) stands for its transpose, A^{-1} represents its inverse and $\mathrm{tr}\{A\}$ denotes its trace.

2. Problem formulations and preliminaries

This section gives the system description, control objective and some related preliminaries.

2.1. Switched nonlinear systems

Consider the switched nonlinear system

$$\dot{x} = f_\sigma(x) + g_\sigma(x)u_\sigma,\ y = h(x), \tag{1}$$

where $x \in \Re^n$, $y \in \Re^q$ are the system state and output, respectively. $u_i \in \Re^p$ is the control input of the ith subsystem. σ: $[0, +\infty) \longrightarrow M = \{1, 2, \ldots, m\}$ is the switching signal with m being the number of subsystems. In addition, $f_i(x)$, $g_{ik}(x)$, $(g_i(x) = [g_{i1}(x), \ldots, g_{ip}(x)])$, $i \in M$, $k = 1, \ldots, p$, are unknown smooth vectors. $h(x)$ is an unknown continuous differentiable function.

Assumption 1. The solution of system (1) are continuous everywhere.

It is important to note that Assumption 1 is a common assumption in the study of switched systems [30,31].

2.2. RBFNNs approximation properties

In recent years, RBFNNs have become more and more popular because of their good capabilities in function approximation. As stated in [32], for an unknown continuous function $F(X)$ defined on $\Omega_X \subset \Re^n$ and arbitrary $\varepsilon > 0$, there exists an integer $W^{*T}\varphi(X)$, such that

$$\sup_{X\in\Omega_X} |W^{*T}\varphi(X) - F(X)| \le \varepsilon,$$

where $X \in \Re^n$ is the input variable, W^* is the ideal weight vector, $\varphi(X) = [\phi_1(X), \ldots, \phi_\tau(X)]^T$ is the smooth basis vector with the RBFNNs node number τ, and $\phi_\mu(X)$, $\mu = 1, \ldots, \tau$, are chosen as the commonly used Gaussian functions

$$\phi_\mu(X) = \exp(-(X - \pi_\mu)^T(X - \pi_\mu)/\upsilon_\mu^2), \quad \mu = 1, \ldots, \tau,$$

where $\pi_\mu = [\pi_{\mu 1}, \ldots, \pi_{\mu n}]^T$ and υ_μ are the centers and the widths of the Gaussian functions, respectively.

Based on the monotonic property of exponential function, we get $0 < \phi_\mu(\cdot) < 1$ which leads to $\phi_\mu^2(\cdot) < 1$. Therefore, the following inequality holds:

$$\varphi^T(X)\varphi(X) \le \tau.$$

According to the approximation property of neural networks, the unknown function $F(X)$ can be written in the following form:

$$F(X) = W^{*T}\varphi(X) + \varepsilon(X),$$

where $\varepsilon(X)$, called approximation error, is usually assumed to be upper bounded with $\bar{\varepsilon}$, i.e., $\|\varepsilon(X)\| \le \bar{\varepsilon}$.

2.3. Prescribed performance

The objective of the prescribed performance is to guarantee that the tracking errors $e_j = y_j - y_{d_j}$, $j = 1, 2, \ldots, q$, are not beyond the specified prescribed performance bounds (PPBs), where y_{d_j} are the given reference signals with known bounds and known bounded derivatives [33]. These PPBs can be described as

$$-\rho_j\varsigma_j(t) \le e_j(t) \le \varsigma_j(t), \quad \text{if } e_j(0) \ge 0, \tag{2}$$

$$-\varsigma_j(t) \le e_j(t) \le \rho_j\varsigma_j(t), \quad \text{if } e_j(0) < 0, \tag{3}$$

where the constants ρ_j, $j = 1, 2, \ldots, q$, satisfy the conditions $0<\rho_j<1$. $\varsigma_j(t)$, $j = 1, 2, \ldots, q$, are the smooth decreasing performance functions which are denoted as

$$\varsigma_j(t) = (\varsigma_{j0} - \varsigma_{j\infty})\exp(-l_jt) + \varsigma_{j\infty}, \tag{4}$$

where ς_{jo} and $\varsigma_{j\infty}$ are positive constants. $\varsigma_{j\infty}$ stand for the maximum amplitudes which impose restriction on the tracking errors to guarantee their steady state and the decreasing rate $\exp(-l_jt)$ confine the convergence speed of tracking errors. The maximum overshoot and undershoot of $e_j(t)$ can be achieved by appropriately choosing the value of $\varsigma_j(0)$ and $\rho_j\varsigma_j(0)$.

Next, we define the transformation errors z_j as

$$z_j = S_j(e_j(t)/\varsigma_j(t)), \quad j = 1, \ldots, q, \tag{5}$$

where $S = [S_1, \ldots, S_q]^T$ is a smooth, strictly increasing and invertible function. We select the function as

$$S(e/\varsigma) = \ln((\rho + e/\varsigma)/(1 - e/\varsigma)), \quad \text{if } e(0) \ge 0, \tag{6}$$

$$S(e/\varsigma) = \ln((1 + e/\varsigma)/(\rho - e/\varsigma)), \quad \text{if } e(0) < 0, \tag{7}$$

with $e = [e_1, \ldots, e_q]^T$, $\rho = [\rho_1, \ldots, \rho_q]^T$ and $\varsigma = [\varsigma_1, \ldots, \varsigma_q]^T$.

Thus, we can obtain the derivative of z as

$$\dot{z} = r(L_{f_\sigma}h(x) + L_{g_\sigma}h(x)u_\sigma - v), \tag{8}$$

where $z = [z_1, \ldots, z_q]^T$, $r = diag[r_1, \ldots, r_q]$ and $v = [v_1, \ldots, v_q]^T$ with $r_j = (1/\varsigma_j)(\partial S_j/\partial(e_j/\varsigma_j))$ and $v_j = \dot{y}_{d_j} + (e_j\dot{\varsigma}_j)/\varsigma_j$, $j = 1, \ldots, q$. Since both r and v are known so that they can be used to design a controller.

Then, the transformed system can be obtained as

$$\begin{cases} \dot{x} = f_\sigma(x) + g_\sigma(x)u_\sigma, \\ \dot{z} = r(L_{f_\sigma}h(x) + L_{g_\sigma}h(x)u_\sigma - v). \end{cases} \tag{9}$$

2.4. Control objective

In this paper, the control objective is to design a controller for each subsystem such that:

(1) the output y tracks a given reference signal $y_d(t)$ to a small compact set;

(2) all the signals in the closed-loop system are bounded;

(3) the tracking error achieves prescribed transient and steady state performance under arbitrary switching signals.

Remark 1. It is worth noting that although there exists switching, the transient performances can also be guaranteed at the switching point.