ARTICLE IN PRESS

Neurocomputing (xxxx) xxxx–xxxx

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Distributed consensus tracking for the fractional-order multi-agent systems based on the sliding mode control method☆

Jing Bai[a,*], Guoguang Wen[b], Ahmed Rahmani[c], Yongguang Yu[b]

[a] School of Mathematics and Physics, University of Science and Technology, Beijing 100083, China
[b] Department of Mathematics, Beijing Jiaotong University, Beijing 100044, PR China
[c] CRIStAL, UMR CNRS 9189, Ecole Centrale de Lille, Villeneuve d'Ascq 59651, France

ARTICLE INFO

Communicated by Marco Loog

Keywords:
Distributed consensus tracking
Lyapunov candidate function
Fractional-order
Sliding mode control

ABSTRACT

Distributed consensus tracking for the fractional-order multi-agent systems is mainly studied in this paper. Firstly, the simple Lyapunov candidate function is discussed to judge the validity of the proposed controller. Secondly, according to the sliding mode control method, a controller is designed to achieve the consensus tracking problem when the followers are described by the fractional-order linear dynamics. Thirdly, the case when the dynamics of followers own the intrinsic nonlinear function is discussed, it proves that the designed sliding mode controller is still valid for this case under the certain conditions. For the above two parts, the systems stabilities are judged based on the result of the first part. Finally, several simulations are presented to verify the obtained results.

1. Introduction

Distributed coordination control means that agents work in a cooperative fashion through decentralized controllers with local information and limited inter-agent communication. In recent year, the study on distributed coordination control has drawn great attention due to its advantages, such as low operational costs, high robustness, flexible scalability and so on. Consensus plays an important role in the study of distributed coordination control due to its broad applications, which include sensor networks, multi-robots, multiple satellites and so on [1–4].

As one important part, consensus tracking has become very popular in many fields, such as biotic population, formation flocking, body guard and so on [5]. Consensus tracking represents that all followers can asymptotically track one leader (or a group of leaders). Up to now, many results have been given on the consensus tracking problem based on the integer-order systems. For instance, the problem has been studied from single-integrator systems to double-integrator systems [6–8]. In addition, various controllers have been designed according to different methods [9–11]. Moreover, uncertain models, time-delay, external noisy have been widely investigated due to the existence of external disturbances when agents work in different environments [12–14]. However, lots of systems cannot be described by integer-order systems when the work environments for agents are complex, for example, the undersea with a large number of viscous substances or microorganisms for the underwater robots, the complex space with lots of particles for the unmanned aerial vehicles and so on [15]. While fractional-order systems show the systems characteristics very well in various fields due to its powerful memory and hereditary properties [5,16–18].

Because of the potential applications of the fractional-order systems in the coordination control, the fractional-order multi-agent systems has been studied since 2008 in [15], where the key focus is the consensus problem. Then, researchers have focused on the study of consensus producing without a leader [19,20], at the same time, external disturbances, time delay, uncertain systems have also been considered [21–23]. Moreover, various control approaches have been given to investigate the consensus problem, which contain fractional-order PID control [19], adaptive control [24], event triggering control [25] and so on. For the control approaches, the sliding mode control method is well known for its robustness against system external disturbance and model uncertainty, and the method can realizes system control fast. It has been widely used to study the control problem both for integer-order systems [26] and fractional-order systems [27,28]. For example, due to the additional design parameters for tuning, fractional-order sliding mode method has been developed

☆ This work was supported by the Fundamental Research Funds for the Central Universities under the University of Science and Technology Beijing under Grant FRF-TP-16-011A1, and the National Natural Science Foundation of China under Grants 61403019 and 11371049.

* Corresponding author.
E-mail address: baijing_good@126.com (J. Bai).

http://dx.doi.org/10.1016/j.neucom.2016.12.066
Received 19 May 2016; Received in revised form 2 December 2016; Accepted 24 December 2016
0925-2312/

for wind energy systems,lighting control systems, photovoltaic systems and so on. However, in the study of the fractional-order multi-agent systems, there is no corresponding results using the sliding mode control method.

In the study of the fractional-order multi-agent systems, the stability plays a key role. For the fractional-order linear multi-agent systems, different theories have been applied to judge the stability [29,30]. However, because of the complexity of the nonlinear systems, few theories can be used to judge the stability for the fractional-order nonlinear multi-agent systems [31,32], where the Lyapunov direct method for the fractional-order systems has been studied, which provides a valid tool to judge the stability problem, but for this method, the difficulty is to search an adaptive Lyapunov candidate function [33,34].

Based on the above analysis, this paper will discuss the consensus tracking problem for the fractional-order multi-agent systems, and the sliding mode control method is applied to design the effective controller. The main contributions can be presented as follows. Firstly, a simple Lyapunov candidate function is studied, and it proves that the key inequality for the differential of the Lyapunov candidate function is satisfied. Secondly, when the dynamics of followers can be shown by the fractional-order linear systems, it proves that the designed sliding mode controller is valid to achieve the consensus tracking under certain conditions. Thirdly, due to many complex phenomena can be described by the nonlinear dynamics, hence, the case when the dynamics of followers own the intrinsic nonlinear function is investigated, it also proves that the above controller for the consensus tracking is effective. Finally, several simulations are proposed to prove the above results. Compared to the existed references, the differences of this paper can be summarized as follows. Firstly, comparing with the presented discrete Lyapunov candidate functions in [33,24], a classic continuous Lyapunov candidate function for the integer-order systems is extended to solve the stability problem of the fractional-order nonlinear systems. Secondly, due to theory deficiency, few results have been obtained on the distributed coordination of the fractional-order multi-agent systems. Based on the above consideration, the paper designs an adaptive controller for the fractional-order multi-agents systems, and then the distributed consensus tracking can be guaranteed, which means that agents just receive information from their neighbors, and track the leader eventually. Thirdly, different from the designed linear controllers for the fractional-order multi-agent systems in [33,34], the paper proposes an effective nonlinear controller based on the sliding mode control method.

We arrange the rest of the paper as follows. In Section 2, the definition of the Caputo fractional derivative, the Lyapunov direct method for the fractional-order systems and some existed results are presented, respectively. In Section 3, a Lyapunov candidate function is discussed, then, for the fractional-order multi-agent systems, the distributed consensus tracking is investigated by using the sliding mode control method. In Section 4, several simulations are drawn to prove the acquired results. Finally, the paper is concluded in Section 5.

2. Preliminaries

2.1. Fractional calculus

For the fractional-order systems, the Caputo derivative definition holds an important position, since its initial value can show the practical physical significance. Hence, we will use Caputo fractional derivative in the paper, it can be described as follows [5]:

$$ {}_a^C D_t^{\alpha} x(t) = \frac{1}{\Gamma(m-\alpha)} \int_a^t \frac{x^{(m)}(\tau)}{(t-\tau)^{\alpha-m+1}} d\tau, $$

where α represents any real number, m denotes an integer constant, it satisfies $m-1 \le \alpha < m$, ${}_a^C D_t^{\alpha}$ means the Caputo derivative, and α is the system order, and $\Gamma(\cdot)$ represents the Gamma function $\Gamma(p) = \int_0^{+\infty} t^{p-1} e^{-t} dt$, the following property is satisfied:

$$ \Gamma(p+1) = p\Gamma(p), $$

where p denotes any real number.

For Caputo fractional derivative, its Laplace transform plays an important role, which can given as follows:

$$ L\{x^{(\alpha)}(t)\} = s^{\alpha} X(s) - \sum_{k=0}^{m-1} s^{\alpha-k-1} x^{(k)}(0), \tag{1} $$

where $X(s) = L\{x(t)\} = \int_0^{+\infty} e^{-st} x(t) dt$.

Due to $\alpha \in (0, 1]$ is assumed in the paper, the Laplace transform can be rewritten as follows:

$$ L\{x^{(\alpha)}(t)\} = s^{\alpha} X(s) - s^{\alpha-1} x(0). \quad \alpha \in (0, 1] \tag{2} $$

To judge the system stability, the Lyapunov direct method for the integer-order systems was extended for the fractional-order nonlinear systems in [31], which can be called as fractional-order Lyapunov direct method for short.

Lemma 2.1 (*Fractional-order Lyapunov direct method*). *The fractional-order system is called Mittag-Leffler stable at the equilibrium point $\bar{x} = (0, 0, \ldots, 0)^T$ if a continuously differentiable function $V(t, x(t))$ is existed, the following conditions are satisfied:*

$$ \alpha_1 \parallel x \parallel^{c} \le V(t, x(t)) \le \alpha_2 \parallel x \parallel^{cd}, \tag{3} $$

$$ {}_a^C D_t^{\alpha} V(t, x(t)) \le -\alpha_3 \parallel x \parallel^{cd} \tag{4} $$

*where the locally Lipschitz condition on x is satisfied for $V(t, x(t))$: $[0, +\infty) \times D \to R$, $D \subset R^n$ is a domain, which contains the origin; $t \ge 0$, $\alpha \in (0, 1)$, $\alpha_1, \alpha_2, \alpha_3, c$ and d represent any positive constants. $\bar{x} = (0, 0, \ldots, 0)$ is globally Mittag-Leffler stable if the assumptions hold globally on R^n, which implies that $\bar{x}$ is asymptotic stability.*Different from the classic Leibniz rule for the integer-order systems, for the fractional-order systems, the rule can be is described as follows.

Property 1. *If $f(t)$ and $g(t)$ along with all its derivatives are continuous in $[t_0, t]$, then for fractional differentiation, the Leibniz rule takes the form:*

$$ {}_{t_0}^C D_t^{\alpha}(f(t)g(t)) = \sum_{k=0}^{\infty} \binom{\alpha}{k} f^{(k)}(t) {}_{t_0}^C D_t^{\alpha-k} g(t). \tag{5} $$

Note that, unlike the Leibniz rule of the classic integer-order systems, the Leibniz rule for the fractional-order systems is complex, which is not easy to be used for finding an adaptive Lyapunov candidate function. Hence, according to the Lyapunov direct method for the fractional-order systems, we will apply an adaptive Lyapunov candidate function based on the following obtained inequalities [32].

Lemma 2.2. *Let $x(t) \in R$ be a continuous and derivable function. Then, for any time instant $t \ge t_0$*

$$ \frac{1}{2} {}_{t_0}^C D_t^{\alpha} x^2(t) \le x(t) {}_{t_0}^C D_t^{\alpha} x^2 x(t), \ \alpha \in (0, 1] \tag{6} $$

Remark 2.3. Obviously, when $x(t) \in R^n$, the following relation is satisfied:

$$ \frac{1}{2} {}_{t_0}^C D_t^{\alpha}(x^T(t)x(t)) \le x^T(t) {}_{t_0}^C D_t^{\alpha} x(t), \ \alpha \in (0, 1] \tag{7} $$