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Multi-criteria group decision making with incomplete hesitant fuzzy preference relations

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ABSTRACT

In order to simulate the hesitancy and uncertainty associated with impression or vagueness, a decision maker may give her/his judgments by means of hesitant fuzzy preference relations in the process of decision making. The study of their consistency becomes a very important aspect to avoid a misleading solution. This paper defines the concept of additive consistent hesitant fuzzy preference relations. The characterizations of additive consistent hesitant fuzzy preference relations are studied in detail. Owing to the limitations of the experts' professional knowledge and experience, the provided preferences in a hesitant fuzzy preference relation are usually incomplete. Consequently, this paper introduces the concepts of incomplete hesitant fuzzy preference relation, acceptable incomplete hesitant fuzzy preference relation, and additive consistent incomplete hesitant fuzzy preference relation. Then, two estimation procedures are developed to estimate the missing information in an expert's incomplete hesitant fuzzy preference relation. The first procedure is used to construct an additive consistent hesitant fuzzy preference relation from the lowest possible number, $(n-1)$, of pairwise comparisons. The second one is designed for the estimation of missing elements of the acceptable incomplete hesitant fuzzy preference relations with more known judgments. Moreover, an algorithm is given to solve the multi-criteria group decision making problem with incomplete hesitant fuzzy preference relations. Finally, a numerical example is provided to illustrate the solution processes of the developed algorithm and to verify its effectiveness and practicality.

1. Introduction

As a new extension of fuzzy sets [39], Torra [20] proposed the concept of hesitant fuzzy sets (HFSs) to enhance the modeling abilities of fuzzy sets. The core of a hesitant fuzzy set (HFS) is a hesitant fuzzy element (HFE) [26], which consists of several possible values for the membership degree. For example, suppose that a group of decision makers (DMs) are hesitant about some possible values as 0.5, 0.6, and 0.7 to assess the membership of an element x to the set A, and the group of DMs cannot persuade one another to change their own opinions. In such cases, the membership of x to A can be modeled by a HFE represented by $h=\{0.5, 0.6, 0.7\}$, which is different from the situations of using fuzzy sets and its extensions, such as interval-valued fuzzy sets [40], intuitionistic fuzzy sets [2], interval-valued intuitionistic fuzzy sets [3], type-2 fuzzy sets [8], and fuzzy multisets [41]. Due to the advantages of handing imprecision whereby two or more sources of vagueness appear simultaneously [49], HFSs have attracted great attention from scholars and have been widely applied in decision making [15,16,26,34,35,42,43,46–48].

In the process of decision making, the decision maker may feel comfortable to express his/her preferences by comparing each pair of objects and then construct a preference relation. The preference relation, as the most efficient and common representation of information, is composed of a collection of preference values, each of which is provided by an expert to express his/her opinion over a pair of objects by means of a predefined scale. With the different types of scales, many different types of preference relations have been proposed, such as the fuzzy preference relation [14,19], the multiplicative preference relation [17], the linguistic preference relation [7,9], the intuitionistic fuzzy

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preference relation [30,31], the intuitionistic multiplicative preference relation [27], the interval-valued intuitionistic fuzzy preference relation [32,36], the interval-valued intuitionistic multiplicative preference relation [38], the interval fuzzy preference relation [29], the interval multiplicative preference relation [18], the triangular fuzzy preference relation [28], and triangular fuzzy reciprocal preference relation [21]. However, all these preference relations do not consider the hesitant fuzzy information, and thus they cannot provide all the possible evaluation values of the decision makers when comparing pairwise alternatives (or criteria), which is a common situation in our daily life. To solve this drawback, inspired by HFS, Zhu and Xu [45] first gave the definition of hesitant fuzzy preference relations (HFPRs) and then investigated their distinctive properties. Furthermore, Zhu and Xu [45] proposed a regression method to transform hesitant fuzzy preference relations into fuzzy preference relations (FPRs). Moreover, Zhu et al. [49] explored the ranking methods with HFPRs in the group decision making environments. Liao et al. [12] investigated the multiplicative consistency of HFPRs and its application in group decision making.

Owing to the limitations of the experts' professional knowledge and experience, or time pressure, the provided preferences in a HFPR are usually incomplete, especially for the preference relation with high order. That is to say, an expert may be unfamiliar with a certain object and thus unable to provide preferences associated with it, or an expert may be unwilling to express his opinions over some pairs of objects because of emotional factors. In such cases, it would be sensible not to force the expert to express "false" preferences over these objects, and thus an incomplete HFPR could be constructed, in which some elements are missing. As a result, estimation of the missing information in an expert's incomplete HFPR becomes an interesting and important research topic, however, nothing has been done about it. In this paper, we shall focus on solving this issue. To do this, we define the concept of additive consistent HFPRs and examine the characterizations of additive consistent HFPRs. Based on these new characterizations, we have first developed an algorithm for estimating the missing elements using only the known preference values in an acceptable incomplete HFPR with the lowest number of judgments, and then extend it to estimate missing elements of the acceptable incomplete HFPRs with more known judgments. A new algorithm is then laid out for handling multi-criteria group decision making problems with acceptable incomplete HFPRs.

The remainder of this paper is set out as follows. Section 2 presents some basic knowledge about the FPR, the HFS, and the HFPR. Section 3 first introduces the concept of additive consistent HFPRs. The properties of additive consistent HFPRs are then studied in detail. Two approaches to constructing additive consistent HFPRs based on acceptable incomplete HFPRs are proposed in Section 4. After that, Section 5 develops an approach to multi-criteria group decision making based on incomplete HFPRs, and furthermore, a numerical example is given to illustrate the validity and applicability of the proposed method in Section 6. The paper ends with some concluding remarks in Section 7.

2. Preliminaries

In this section, we will briefly recall the concepts of fuzzy preference relation, hesitant fuzzy set, and hesitant fuzzy preference relation.

2.1. Fuzzy preference relation

Definition 2.1 *([19])*. Let $X=\{x_1, x_2, \ldots, x_n\}$ be a set of alternatives, then $R=(r_{ij})_{n\times n}$ is called a fuzzy preference relation (FPR) on $X\times X$ with the following conditions:

$$r_{ij} \geq 0, \quad r_{ij}+r_{ji}=1, \quad i, j=1,2,\ldots,n, \tag{1}$$

where r_{ij} denotes the degree that the alternative x_i is preferred to the alternative x_j provided by the decision maker. Especially, $r_{ij}=0.5$ indicates indifference between x_i and x_j; $r_{ij}>0.5$ indicates x_i is preferred to x_j, the larger the r_{ij}, the greater the preference degree of the alternative x_i over x_j, $r_{ij}=1$ indicates that x_i is absolutely preferred to x_j; $r_{ij}<0.5$ indicates x_j is preferred to x_i; the smaller the r_{ij}, the greater the preference degree of the alternative x_j over x_i, $r_{ij}=0$ indicates that x_j is absolutely preferred to x_i.

Definition 2.2 *([19])*. Let $R=(r_{ij})_{n\times n}$ be a FPR, then $R=(r_{ij})_{n\times n}$ is called an additive consistent FPR if it satisfies the following additive transitivity:

$$r_{ij}=r_{ik}-r_{jk}+0.5 \quad \text{for all } i, j, k=1,2,\ldots,n. \tag{2}$$

2.2. Hesitant fuzzy set

Torra [20] originally proposed the concept of hesitant fuzzy sets to manage the situations in which several values are possible for the definition of the membership of an element.

Definition 2.3 *([20])*. Let X be a reference set, a hesitant fuzzy set (HFS) on X is in terms of a function that when applied to X returns a subset of [0, 1], which can be represented as the following mathematical symbol:

$$A=\{\langle x, h_A(x)\rangle | x\in X\}, \tag{3}$$

where $h_A(x)$ is a set of some values in [0, 1], which denote the possible membership degrees of the element $x\in X$ to the set A. For convenience, Xia and Xu [26] called $h=h_A(x)$ a hesitant fuzzy element (HFE).

Let l_h denote the number of elements in the HFE h. Note that the number of values in different HFEs may be different and these values are usually unordered. Given a HFE $h=\{h^{\sigma(s)}|s=1,2,\ldots,l_h\}$, it is assumed that are all possible values of the HFE h are arranged in an increasing order, and thus $h^{\sigma(s)}$ is the sth smallest value in h.

We also note that different HFEs have different numbers of elements in most cases. In order to operate correctly when comparing two HFEs, Xu and Xia [34] gave the following rule: We can extend the shorter HFE until both of them have the same length by adding the same value to it several times. Zhu et al. [49] introduced a method to add elements in a HFE.