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A simulated annealing-based permutation method and experimental analysis for multiple criteria decision analysis with interval type-2 fuzzy sets

Jih-Chang Wang[a,1], Ting-Yu Chen[b,*]

[a] Department of Information Management, College of Management, Chang Gung University, 259, Wen-Hwa 1st Road, Kwei-Shan, Taoyuan 333, Taiwan

[b] Department of Industrial and Business Management, Graduate Institute of Business and Management, College of Management, Chang Gung University, 259, Wen-Hwa 1st Road, Kwei-Shan, Taoyuan 333, Taiwan

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ABSTRACT

The aim of this paper is to develop a simulated annealing-based permutation method for multiple criteria decision analysis within the environment of interval type-2 fuzzy sets. The outranking methodology constitutes one of the most fruitful approaches in multiple criteria decision making and has been applied in numerous real-world problems. The permutation method is a classical outranking model, which generalizes Jacquet–Lagreze's permutation method and is based on a pairwise criterion comparison of the alternatives. Because modeling of the uncertainty in the decision-making process becomes increasingly important, an extension to the interval type-2 fuzzy environment is a useful generalization of the permutation method and is appropriate for handling uncertain and imprecise information in practical decision-making situations. This paper produces a signed-distance-based comparison among the comprehensive rankings of alternatives for concordance and discordance analyses. An integrated nonlinear programming model is constructed for estimation of the criterion weights and the optimal ranking order of the alternatives under incomplete preference information. To enhance the implementation efficiency, a simulated annealing-based permutation method and its meta-heuristic algorithm are developed to produce a polynomial time solution for the total completion time problem. Furthermore, computational experiments with notably large amounts of simulation data are conducted to test the solution approach and validate the correctness of the approximate solution compared with the optimal all-permutation-based result.

1. Introduction

Multiple criteria decision analysis (MCDA) problems address the ranking of alternatives and the selection of the best alternative among a finite set of alternatives based on a finite set of criteria with incomplete information [1,2]. The outranking methodology constitutes one of the most fruitful approaches in MCDA and has been applied in many real-life problems [3]. Specifically, the outranking model arranges a set of preference rankings that best satisfy a given concordance measure [4]. In particular, the permutation method from Paelinck's qualitative multiple criteria analysis [5,6] is a classical outranking model in MCDA. This method, which generalizes Jacquet–Lagreze's permutation method, is a metric procedure and is based on an evaluation of all possible rankings (permutations) of the alternatives under consideration [3]. Uncertain and imprecise assessment of information often occurs in practical decision-making situations [7,8]. Thus, modeling of uncertainty in subjective judgments and evaluation processes becomes increasingly important in handling MCDA problems [9,10]. Accordingly, an extension to the fuzzy environment is a natural generalization of the permutation-based outranking methodology. For example, Chen et al. [11] developed an intuitionistic fuzzy permutation method for addressing MCDA problems. Chen and Wang [12] proposed an interval-valued fuzzy permutation method and conducted an experimental analysis using cardinal and ordinal evaluations.

Nevertheless, available information is sometimes not sufficient for the exact definition of a degree of membership for certain elements [7]. It is not reasonable to use an accurate membership

* Corresponding author. Tel.: +886 3 2118800x5678; fax: +886 3 2118500.
E-mail addresses: qpo@mail.cgu.edu.tw (J.-C. Wang), tychen@mail.cgu.edu.tw (T.-Y. Chen).
[1] Tel.: +886 3 2118800x5821; fax: +886 3 2118700.

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function for a factor that is uncertain [13–15]. Therefore, type-1 fuzzy sets (T1 FSs) cannot fully address all of the uncertainty present in real-world problems [16]. The concept of type-2 fuzzy sets (T2 FSs) is an extension of T1 FSs [17,18]. T2 FSs are superior to T1 FSs because T2 FSs can model second-order uncertainties [19,20]. Unfortunately, the computational complexity of using T2 FSs is very high, which makes it very difficult to employ them in practical applications [21,22] (Zhang, 2013). Due to the high computational complexity of using T2 FSs, interval type-2 fuzzy sets (IT2 FSs) have become the most widely used T2 FSs [23–28]. This paper attempts to focus on the permutation-based outranking methodology within the decision environment of IT2 FSs.

In addition to considering the context of IT2 FSs, Chen et al. [29] developed an extended qualitative flexible multiple criteria method (QUALIFLEX) for handling medical decision-making problems. Wang et al. [30] introduced a likelihood-based QUALIFLEX method for addressing MCDA problems. Both the extended QUALIFLEX method and the likelihood-based QUALIFLEX method belong to the permutation-based outranking methodology based on IT2 FSs. Although the feasibility and effectiveness of the above-mentioned methods in the MCDA field have been thoroughly investigated and demonstrated, the main drawback of these methods is low computation efficiency in cases of numerous alternatives.

In fact, the existing permutation-based outranking methods have an evident limitation because the solution for all of the possible permutations is an NP-hard problem. The permutation methods need to list all of the possible permutations of the alternatives. If the number of alternatives is not very large, for example, seven and ten alternatives, it would need to list 5040 and 3,628,800 permutations, respectively. It seems that a simple problem would be a complicated problem when the permutation methods are used. Furthermore, this paper has conducted a pilot study to investigate the average CPU time for generating all possible permutations of the alternatives. For each number of alternatives, ten trials were implemented using the MATLAB run on an x64-based PC with an Intel Core i7-4500U (1.8 GHz) CPU, 8 G RAM, and an operating system Windows 7. The average CPU times for producing all permutations are 0.000953, 0.003609, 0.018750, 0.109375, 0.76875, 6.16875, 55.5, 556.23 (i.e., 9.27 min), and 6100 (i.e., 1.69 h) s for cases with three to eleven alternatives, respectively. Obviously, the average CPU times in the cases of ten and eleven alternatives are significantly high. In general, the number of permutations increases rapidly with an increasing number of alternatives. It directly follows that the computational complexity of the existing permutation-based outranking methods is evidently high in the case of many alternatives. Therefore, current methods will encounter great difficulties in a large-size decision-making problem. Moreover, this problem will be more serious within the IT2 FS environment because of sophisticated operations based on type-2 fuzzy logic. For the above reasons, this paper intends to develop a new permutation-based method in the context of IT2 FSs to overcome the difficulty of huge computations and to capture imprecise and uncertain information.

Simulated annealing (SA) has been widely applied in NP-hard problems [31]. The SA is a meta-heuristic optimization method used to solve complex problems with large solution spaces and produces results close to the global optimum value in a short period of time [32]. This algorithm is well suited to solving large-scale and difficult optimization problems [33,34]. The concept of SA is inspired by nature and is derived from statistical mechanics [32]. The SA method emulates the solid annealing process, which first heats a solid to its melting point and subsequently slowly cools the material [35]. Specifically, the SA method is based on an analogy with thermodynamics and the manner in which liquids freeze and crystallize; moreover, the freezing process is guided by a cooling schedule that controls the decay of the system temperature [34]. The SA method has the advantage in that it avoids becoming trapped in local optima [36]. Thus, the SA method has become one of the most popular meta-heuristic methods and has been applied widely to solve many combinatorial optimization problems [35–37]. SA provides a suitable approach to solving the optimization problem in a wide range of applications [38,39] and is a stochastic optimization technique that converges on the global optimal solution. Therefore, this paper attempts to develop an SA-based permutation method to improve the computation efficiency and acquire a polynomial time solution.

The purpose of this paper is to construct a new outranking approach, i.e., an SA-based permutation method, for solving MCDA problems under incomplete preference information within the environment of IT2 FSs. In the context of interval type-2 trapezoidal fuzzy numbers (IT2 TrFNs), this paper proposes the main structure of the permutation-based outranking method for addressing the problems caused by greater imprecision or uncertainty in MCDA. Instead of relying on a complicated computational process to handle IT2 TrFN data, this paper develops a simple and effective comparative method based on the concept of signed distances to differentiate the sets of concordance and discordance. The proposed method offers a flexible approach capable of tackling decision-making problems featuring conflicting information with respect to criterion importance under the incomplete preference structure. Furthermore, this paper provides an SA-based permutation algorithm for the solution approach to enhance the implementation efficiency of the proposed method, especially in case of many alternatives, and determines a polynomial time solution for the total completion time problem.

This paper makes several significant contributions to the existing literature on the permutation-based outranking methodology in the MCDA field. First, we establish the fundamental structure of the SA-based permutation method based on IT2 TrFNs to address second-order uncertainties in decision reality. Second, compared to other existing permutation methods, the proposed method provides a flexible approach capable of manipulating incompletely known or even conflicting information about criterion importance in practice. Third, the proposed method uses a hybrid approach that integrates SA into the permutation-based outranking methodology to adapt to the MCDA problems within the interval type-2 fuzzy environment. Fourth, the difficulty of implementing computations for MCDA problems with numerous alternatives can be significantly overcome with the help of the proposed SA-based permutation algorithm. Fifth, the applicable scope of the problem size can be evidently expanded because the SA-based permutation algorithm can improve the computation efficiency and acquire a polynomial time solution. In contrast, the existing permutation methods can only be applied to MCDA problems with a limited number of alternatives. Finally, the computational experiments and the comparative analysis validate the effectiveness and efficiency of the proposed method for applications. Thus, the proposed SA-based permutation method not only improves the established methods but also enriches the development of various permutation-based outranking methods, especially within the decision environment of IT2 FSs.

This paper is organized as follows. Section 2 briefly reviews the basic concepts of T2 FSs, IT2 FSs, and IT2 TrFNs. Section 3 formulates an MCDA problem in the IT2 TrFN framework and develops the SA-based permutation method under incomplete preference information. Section 4 conducts a comparative analysis via computational experiments to examine the effectiveness and efficiency of the proposed method. This section also consists of comparisons with other relevant methods and discusses the advantages of the proposed method. Finally, Section 5 presents the conclusions.