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Pinning synchronization of spatial diffusion coupled reaction-diffusion neural networks with and without multiple time-varying delays

Shu-Xue Wang[a], Yan-Li Huang[a,b,*], Bei-Bei Xu[a]

[a] *School of Computer Science and Software Engineering, Tianjin Polytechnic University, Tianjin 300387, China*
[b] *Tianjin Key Laboratory of Optoelectronic Detection Technology and System, Tianjin Polytechnic University, Tianjin 300387, China*

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ABSTRACT

In this paper, two coupled reaction-diffusion neural networks (CRDNNs) with spatial diffusion coupling are studied. In the first one, the single reaction-diffusion neural network (RDNN) is coupled by their current states. The single RDNN is coupled by their current states and delayed states in the second one. Combined with some inequality techniques and Lyapunov functional approach, a synchronization criterion for the first network model is established via adding controllers to the first l nodes. In addition, a sufficient condition is derived to make sure that the considered network can achieve synchronization by designing pinning adaptive feedback controllers. Similarly, the pinning synchronization for the second network model is also considered. Finally, the correctness of the obtained results are confirmed by numerical simulation in two illustrated examples.

1. Introduction

Recently, more and more attention has been paid to neural networks (NNs) because of their extensive applications, such as pattern recognition, associative memory, optimization, signal processing and other engineering or scientific fields [1,2]. Thus, the dynamical behaviors of NNs were investigated by many researchers. In [3], Ahn studied the exponential filter and passive for Takagi-Sugeno fuzzy Hopfield NNs. Yang et al. [4] considered the problem of stability for NNs.

In the last few years, research into the applications of coupled neural networks (CNNs) have developed very fast in science and engineering. In many situations, the applications of CNNs mainly depend on their dynamical behaviors [5–10], especially, the synchronization of CNNs. Hence, many scholars have considered the synchronization of CNNs [11–20]. In [11], the authors discussed impulsive synchronization for a CNNs at partly unknown transition probabilities. Zhang et al. [13] focused on the synchronization and stability of memristor-based CNNs. In [14], the authors paid attention to the mean square synchronization of coupled stochastic NNs with on-off coupling which is periodic.

Actually, diffusion phenomena can't be avoided in NNs [21–29]. So far, a great deal of researchers have investigated the dynamical behaviors of reaction-diffusion neural networks (RDNNs). In [21], some sufficient conditions were derived to make an impulsive Cohen-Grossberg RDNNs with delays realize global exponential stability. The robust global exponential stability was discussed for interval Hopfield RDNNs [25]. The passivity and stability problems of RDNNs were studied in [24]. However, very few research results on the synchronization for coupled RDNNs (CRDNNs) have been got [26–29]. In [26], Wang and Wu considered synchronization and $\mathcal{H}_\infty$ synchronization of CRDNNs with hybrid coupling. An edge-based adaptive strategy was designed to tune a small number of the coupling strengths which leads to the synchronization of CRDNNs [27]. The authors gave several adaptive synchronization criteria for CRDNNs in [28].

Nevertheless, it is impossible for CNNs to realize synchronization by themselves in many practical situations. Therefore, some control schemes should be designed for synchronization of CNNs. Due to greatly reducing the number of controlled nodes, a very effective strategy called pinning control method is proposed for synchronization of CNNs by adding controllers to a part of nodes [30–34]. In [30], the adaptive exponential synchronization problem of neutral-type CNNs with Markovian switching parameters was investigated by pinning control. Zheng et al. [31] studied robust synchronization of the CNNs by utilizing intermittent pinning control method. As is well-known, there are very few works on pinning synchronization of CRDNNs [35–38]. In [36], Liu et al. obtained some sufficient conditions for pinning global μ-synchronization of CRDNNs. Wang et al. [37] proposed a network model for CRDNNs with directed topology, and several criteria on synchronization were derived for the proposed network model

* Corresponding author at: Tianjin Key Laboratory of Optoelectronic Detection Technology and System, School of Computer Science and Software Engineering, Tianjin Polytechnic University, Tianjin 300387, China.
E-mail address: huangyanli@tjpu.edu.cn (Y.-L. Huang).

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under pinning control technique.

Motivated by previous discussion, we devote to studying the synchronization of spatial diffusion CRDNNs with and without multiple time-varying delays by pinning control method in this paper. The distinctive contributions are as follows. Firstly, we respectively establish two synchronization criteria for spatial diffusion CRDNNs with and without multiple time-varying delays by selecting a fraction of nodes to be pinned with negative feedback controllers. Second, some adaptive strategies to update the pinning feedback gains are designed for reaching synchronization.

2. Preliminaries

2.1. Notations

Let $\mathbb{R} = (-\infty, +\infty)$. $0 \le P \in \mathbb{R}^{n\times n}$ $(0 > P \in \mathbb{R}^{n\times n}, 0 < P \in \mathbb{R}^{n\times n}, 0 \ge P \in \mathbb{R}^{n\times n})$ denotes that matrix P is symmetric and semi-positive (negative, positive, semi-negative) definite. $\Omega = \{x = (x_1, x_2, \ldots, x_m)^T \| \, |x_k| < l_k,\ k = 1, 2, \ldots, m\}$ represents an open bounded domain in $\mathbb{R}^m$ with smooth boundary $\partial\Omega$. $\lambda_m(\cdot)$ $(\lambda_M(\cdot))$ means the minimum (maximum) eigenvalue of the corresponding matrix.

2.2. Some useful lemmas

Lemma 2.1 (*see* [31]). *For any vectors $\alpha_1, \alpha_2 \in \mathbb{R}^n$ and positive matrix $Q \in \mathbb{R}^{n\times n}$, the following matrix inequality holds*:

$$2\alpha_1^T \alpha_2 \le \alpha_1^T Q \alpha_1 + \alpha_2^T Q^{-1} \alpha_2.$$

Lemma 2.2 (*see* [39]). *Let $\Omega = \{x = (x_1, x_2, \ldots, x_m)^T \| \, |x_k| < l_k,\ k = 1,2,\ldots,m\}$ and $g(x) \in C^1(\Omega)$ be a real-valued function which satisfies $g(x)|_{\partial\Omega} = 0$. Then*

$$\int_\Omega g^2(x)dx \le l_k^2 \int_\Omega \left(\frac{\partial g}{\partial x_k}\right)^2 dx.$$

Lemma 2.3 (*see* [40]). *Let $\alpha \in \mathbb{R}$, A_1, A_2, A_3, A_4 be matrices with suitable dimensions. Then the Kronecker product (denoted by $\otimes$) has the following properties*:

(1) $(\alpha A_1) \otimes A_2 = A_1 \otimes (\alpha A_2)$;

(2) $(A_1 \otimes A_2)(A_3 \otimes A_4) = (A_1 A_3) \otimes (A_2 A_4)$;

(3) $(A_1 \otimes A_2)^T = A_1^T \otimes A_2^T$;

(4) $(A_1 + A_2) \otimes A_3 = A_1 \otimes A_3 + A_2 \otimes A_3$.

3. Pinning control of spatial diffusion CRDNNs

3.1. Network model

A single RDNN is presented by

$$\frac{\partial w_i(x,t)}{\partial t} = \sum_{j=1}^{n} b_{ij} f_j(w_j(x,t)) + \sum_{k=1}^{m} \frac{\partial}{\partial x_k}\left(d_{ik}\frac{\partial w_i(x,t)}{\partial x_k}\right) + J_i - a_i w_i(x,t), \quad i = 1, 2, \ldots, n, \tag{1}$$

where J_i corresponds to the constant external input; $f_j(\cdot)$ represents the activation function of the j-th neuron; $w_i(x, t) \in \mathbb{R}$ is the state of the i-th neuron at time t and in space x; b_{ij} stands for the weight of neuron interconnections; $a_i > 0$ is the rate with which the i-th neuron will reset its potential to the resting state when disconnected from the network and external input; $x = (x_1, x_2, \ldots, x_m)^T \in \Omega \subset \mathbb{R}^m$; $d_{ik} > 0$ denotes the transmission diffusion coefficient along the i-th neuron; n is the number of neurons in the network.

The system (1) satisfies the following two conditions:

$$w_i(x, 0) = \phi_i(x), \quad x \in \Omega, \tag{2}$$

$$w_i(x, t) = 0, \quad (x, t) \in \partial\Omega \times [0, +\infty), \tag{3}$$

where $\phi_1(x), \phi_2(x), \ldots, \phi_n(x)$ are continuous and bounded on Ω.

The following assumption will be needed throughout the paper.

A1) The neuron activation function $f_j(\cdot)$ satisfies the Lipschitz condition, i.e., for any $\zeta_1, \zeta_2 \in \mathbb{R}$, there exists constant $\rho_j > 0$ such that

$$|f_j(\zeta_1) - f_j(\zeta_2)| \le \rho_j |\zeta_1 - \zeta_2|, \ j = 1, 2, \ldots, n,$$

where $|\cdot|$ indicates the Euclidean norm.

By rewriting the system (1), we can get

$$\frac{\partial w(x,t)}{\partial t} = \sum_{k=1}^{m} D_k\left(\frac{\partial^2 w(x,t)}{\partial x_k^2}\right) + Bf(w(x,t)) + J - Aw(x,t), \tag{4}$$

where

$f(w(x,t)) = (f_1(w_1(x,t)), f_2(w_2(x,t)), \ldots, f_n(w_n(x,t)))^T$, $B = (b_{ij})_{n\times n}$, $J = (J_1, J_2, \ldots, J_n)^T$, $w(x,t) = (w_1(x,t), w_2(x,t), \ldots, w_n(x,t))^T$, $D_k = \mathrm{diag}(d_{1k}, d_{2k}, \ldots, d_{nk})$, $A = \mathrm{diag}(a_1, a_2, \ldots, a_n)$.

N identical nodes (4) are coupled into a CRDNNs with spatial diffusion coupling which is shown as:

$$\frac{\partial z_i(x,t)}{\partial t} = \sum_{k=1}^{m} D_k\left(\frac{\partial^2 z_i(x,t)}{\partial x_k^2}\right) + c\sum_{j=1}^{N} G_{ij}\Gamma \triangle z_j(x,t) + Bf(z_i(x,t)) + J - Az_i(x,t), \quad i = 1, 2, \ldots, N, \tag{5}$$

where $\Gamma = (\gamma_{ij})_{n\times n}$ corresponds to the inner coupling matrix; $z_i(x,t) = (z_{i1}(x,t), z_{i2}(x,t), \ldots, z_{in}(x,t))^T \in \mathbb{R}^n$ is the state vector of node i at time t and in space x; $G = (G_{ij})_{N\times N}$ stands for the coupling configuration matrix which has the following definition:

$$\begin{cases} G_{ij} > 0, \text{ if there is a connection from node } i \text{ to node } j\,(i \ne j), \\ G_{ij} = 0, \text{ otherwise } (i \ne j), \end{cases}$$

and

$$G_{ii} = -\sum_{j=1,\, j\ne i}^{N} G_{ij}, \quad i = 1, 2, \ldots, N;$$

$0 < c \in \mathbb{R}$ represents the overall coupling strength; N denotes the number of nodes in the network.

The network (5) satisfies the following conditions:

$$z_i(x,t) = 0, \qquad (x,t) \in \partial\Omega \times [0, +\infty), \tag{6}$$

$$z_i(x, 0) = \Phi_i(x) \in \mathbb{R}^n, \quad x \in \Omega, \tag{7}$$

where $\Phi_1(x), \Phi_2(x), \ldots, \Phi_N(x)$ are continuous and bounded functions on Ω.

For system (1), suppose that $\widehat{w}(x,t)$ is an arbitrary solution, then it satisfies (3) and

$$\frac{\partial \widehat{w}(x,t)}{\partial t} = \sum_{k=1}^{m} D_k\left(\frac{\partial^2 \widehat{w}(x,t)}{\partial x_k^2}\right) + Bf(\widehat{w}(x,t)) + J - A\widehat{w}(x,t),$$

where $\widehat{w}(x,t) = (\widehat{w}_1(x,t), \widehat{w}_2(x,t), \ldots, \widehat{w}_n(x,t))^T$.

Throughout this paper, the network is said to achieve synchronization via pinning control schemes if

$$\lim_{t\to+\infty} \| z_i(\cdot,t) - \widehat{w}(\cdot,t)\|_2 = 0, \quad \text{for all} \quad i = 1, 2, \ldots, N.$$

Remark 3.1. Most of the published results about synchronization of CRDNNs only focused on their state coupling [36–38]. However, very few scholars have devoted to studying the synchronization of CRDNNs with spatial diffusion coupling [35]. It is known to us that different diffusion of each RDNN may have an important effect on other RDNNs in CRDNNs. Hence, it is essential to consider synchronization of spatial diffusion CRDNNs.