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Tracking analysis of coupled continuous networks based on discontinuous iterative learning control☆

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ABSTRACT

The purpose of this paper is to discuss the tracking problem of coupled continuous networks by applying the discontinuous iterative learning control (ILC) method. Two discontinuous ILC schemes with impulsive and zero-order hold (ZOH) sampled-data are introduced because of considering the instantaneous perturbations and the difficulties of measuring the continuous information transmission. Then, some criteria are then proposed to resolve the tracking problem of continuous networks in a finite interval by using the Bellman–Gronwall Lemma. Finally, simulation results are shown to illustrate the effectiveness of the obtained criteria.

1. Introduction

As an effective control strategy, iterative learning control (ILC) has been known for the ability to achieving a perfect tracking of the desired target, which has a long research history as seen from [1–6]. ILC is a well-established technique for improving the performance of a controlled dynamic system that is operated to repeatedly follow the same finite-length desired trajectory over-and-over. It is shown in [7] that multi-agent systems using an iterative learning update algorithm based on the nearest neighbor relative distances could achieve a desired coordinated formation control task under a very relaxed condition that only needs the interaction topology graphs for agents to jointly have a spanning tree over iteration intervals with finite lengths. In [8], an adaptive iterative learning control scheme is proposed for a class of non-linearly parameterised systems with unknown time-varying parameters and input saturations. By using an adaptive iterative learning control approach, Li et al. in [9] discuss the consensus and formation control of distributed multiagent systems with second-order dynamics and unknown time-varying parameters.

There has been a lot of interest in the tracking problem of dynamical systems due to its potential applications in spacecraft formation flying, sensor networks, formation control and so forth. A great deal of research results about the tracking problem have been available (see [10–14] and the references therein). Cao et al. in [15] analyze the condition on the communication graph, the sampling period, and the control gain to ensure stability and show the quantitative bound of the tracking errors. The objective in [16] is to study a distributed coordinated tracking problem for multiple networked Euler–Lagrange systems. In [17], Demigha et al. are interested in collaborative target tracking instead of single node tracking.

Note that the tracking problem is always discussed when time tends to infinity in the above literature. However, in many practical applications such as batch reactor processes, wafer manufacturing processes, IC welding processes and so on, one needs to achieve the desired tracking in a finite-time interval. As a result, the tracking problems have been researched in a finite-time interval in [18–21]. Meanwhile, in these results, the tracking protocols are related to the sign function, which is difficult to be applied in real-world applications. Hence, it is meaningful to introduce ILC in discussing the tracking problem of continuous networks in a finite-time interval. In the most existing results such as [5,6], ILC has always been applied in discrete-time systems. Hence, two challenging questions about ILC arise: (1) How to design the appropriate iterative learning controllers for continuous systems? (2) What kinds of ILC are more reasonable if the realistic networks have to consider instantaneous perturbations and experiences abrupt change at uncertain instants (i.e., impulsive effects [22])?

In order to answer the above two questions, two different ILCs are designed in this paper. The first one is to consider impulsive phenomenon into the ILC. It is worthwhile to note that few work of coupled networks has been done on the ILC scheme with impulsive control. The

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second one is to consider sampled-data control method into the ILC. In the real application, it is quite difficult to obtain the continuous information about time t because of the unreliability of information channels, the capability of transmission bandwidth, etc. Therefore, it is necessary and significative to apply sampled-data control into the ILC. In recent years, sampled-data control has been recently studied and applied in various real-world networks [23–28]. However, in these literature, most learning controllers have been continuous about time t, and few work has been done on the ILC scheme with zero-order hold (ZOH) sampled-data control.

According to the above discussions, the contributions of this study are twofold: (1) The impulsive and ZOH sampled-data ILCs are designed to control coupled continuous networks. Here, the ZOH sampled-data ILC means that the latest control signal will be used until the next new signal is available. (2) Based on the impulsive and ZOH sampled-data ILCs, two criteria are presented to obtain the desired tracking in a finite interval.

The remainder of this paper is: The problem description and the impulsive and ZOH sampled-data ILCs are presented in Section 2. By applying impulsive and ZOH sampled-data ILCs, the tracking problems of continuous networks are discussed in Section 3. In Section 4, simulations are carried out to verify the efficiency of the main results. Finally, conclusions are drawn in Section 5.

Notation: Throughout this study, the superscript T represents the transpose. For all $x = (x_1, x_2, \ldots, x_n)^T \in \mathbb{R}^n$, define $\| x \| = (\sum_{i=1}^{n} x_i^2)^{\frac{1}{2}}$. For a symmetric matrix A, $\lambda_m(A)$ and $\lambda_M(A)$ denote the minimal and maximal eigenvalues of matrix A, respectively. $\| A \|$ denotes the spectral norm defined by $\| A \| = (\lambda_M(A^TA))^{\frac{1}{2}}$. For real symmetric matrixes X and Y, $X > Y$ (or $X \geq Y$) means that matrix $X - Y$ is positive definite (or positive semi-definite).

2. Preliminaries

Consider a n-dimensional coupled continuous network (here, $n > 0$ is an integer). The state vector of the ith node is $x_i(t) = (x_i^1(t), x_i^2(t), \ldots, x_i^N(t))^T \in \mathbb{R}^N$, $i = 1, 2, \ldots, n$, and N is a positive integer, which is the dimension of $x_i(t)$. The network model is

$$\dot{x}_i(t) = f_i(x_i(t)) + \sum_{j=1}^{n} a_{ij}x_j(t), \quad i = 1, 2, \ldots, n, \tag{1}$$

where $t \in [0, T]$ is the fixed time interval with $T > 0$, $f_i(x_i(t, k)) = r_i(t)x_i(t, k) \in \mathbb{R}^N$ is a continuous function, and $A = (a_{ij})_{n\times n}$ is the coupling configuration of the network. If there is a directed connection from node j to node i (here, $i \neq j$, $i, j \in \{1, 2, \ldots, n\}$), then $a_{ij} > 0$; otherwise, $a_{ij} = 0$. Moreover,

$$a_{ii} = -\sum_{j=1, j\neq i}^{n} a_{ij}, \quad i = 1, 2, \ldots, n.$$

As mentioned before, the objective is to apply the ILC method to discuss the tracking of system (1) in a finite time under a repeatable control environment. Moreover, for system (1), we will design two kinds of ILCs (the impulsive and ZOH sampled-data ILCs) due to considering the instantaneous perturbations and the difficulties of measuring the continuous information transmission.

Consider two independent dynamic processes for the iterative learning controllers: the first one describes the dynamics about time t; the second one shows the dynamics about the iterative learning k. Hence, the system with the impulsive and ZOH sampled-data ILCs can be described respectively as

$$\frac{dx_i(t, k)}{dt} = r_i(t)x_i(t, k) + \sum_{j=1}^{n} a_{ij}x_j(t, k) - \delta(t - t_l)u_i(t_l, k), \tag{2}$$

and

$$\frac{dx_i(t, k)}{dt} = r_i(t)x_i(t, k) + \sum_{j=1}^{n} a_{ij}x_j(t, k) - u_i(t, k), \tag{3}$$

where $\delta(s) = \begin{cases} +\infty & s = 0 \\ 0 & s \neq 0 \end{cases}$ is the delta function, $i = 1, 2, \ldots, n$, $l, k \in \mathbb{Z}_+$ ($\mathbb{Z}_+ = \{0, 1, 2, 3, \ldots\}$). For model (2), every node receives the impulsive ILC information at the time sequence $t_0, t_1, \ldots, t_l, \ldots$, which is denoted by $\{t_l\}$. In system (3), the ZOH sampled-data ILC $u_i(t, k)$ only updates its information at the time sequence $\{t_l\}$. That is, $u_i(t, k) = u_i(t_l, k)$, $\forall\, t \in [t_l, t_{l+1})$, which yields that $u_i(t, k)$ in system (3) is called the ZOH sampled-data ILC.

Remark 1. The difference between the impulsive and ZOH sampled-data ILCs is: the impulsive ILC is instantaneous, which only occurs at the time points $t_0, t_1, \ldots, t_l, \ldots$; however, the ZOH sampled-data ILC $u_i(t, k) = u_i(t_l, k)$, $\forall\, t \in [t_l, t_{l+1})$, which is persistent and only updates its information at the time sequence $\{t_l\}$.

Define the desired trajectory of $x_i(t, k)$ as $x_i^r(t)$, and suppose that $x_i^r(t)$ satisfies

$$\dot{x}_i^r(t) = r_i(t)x_i^r(t) + \sum_{j=1}^{n} a_{ij}x_i^r(t), \tag{4}$$

where $l, k \in \mathbb{Z}_+$, $i = 1, 2, \ldots, n$. The boundary conditions of models (2)–(4) are $x_i(0, k) = x_i^r(0) = x_0$, $k \in \mathbb{Z}_+$; $u_i(t, 0) = u_{i0}(t)$, and $\| u_{i0}(t) \| \leq \varepsilon_i$ (ε_i is a positive scalar), $\forall\, i = 1, 2, \ldots, n$, $\forall\, t \in [0, T]$. The tracking error is introduced as

$$e_i(t, k) = x_i^r(t) - x_i(t, k).$$

In the following, the objective is to design the impulsive and ZOH sampled-data ILCs $u_i(t_l, k)$ and $u_i(t, k)$ to obtain $e_i(t, k) \to 0$, $\forall\, t \in [0, T]$, $i = 1, 2, \ldots, n$, as $k \to +\infty$.

Remark 2. Compared with the ILC methods in [7,9], one of the advantages is that the controllers $u_i(t)(i = 1, 2, \ldots, n)$ in systems (2) and (3) only receive or update the impulsive or sampled-data ILC message at the sequence $\{t_l\}(l \in \mathbb{Z}_+)$, rather than receiving the continuous information at any time t. Clearly, it is not economic and practical to transmit the controller $u_i(t)$'s feedback information persistently because of the capability of bandwidth, the unreliability of channels, etc.

In the interval $[0, T]$, the times that $u_i(t_l, k), \forall\, i \in \{1, 2, \ldots, n\}$, update its information must be limited. Hence, we also give the following assumptions.

Assumption 1. $t_0 = 0$, $\forall\, i = 1, 2, \ldots, n$.

Assumption 2. $t_l > t_{l-1}$, for $l \in \{1, 2, \ldots, \Pi\}$, and $t_\Pi = T$, $\Pi > 0$ is an integer, $\forall\, i = 1, 2, \ldots, n$.

Assumption 3. $\forall\, l \in \{0, 1, 2, \ldots, \Pi - 1\}$, $0 < t_{l+1} - t_l \leq \varpi$, where $\varpi \in \mathbb{R}$.

A lemma is needed in the following development.

Lemma 1 ([29] (Bellman-Gronwall Lemma)). *Let $u(t)$ and $f(t)$ be nonnegative, continuous functions on $I = [0, +\infty)$ for which the inequality*

$$u(t) \leq u_0 + \int_0^t f(s)u(s)ds, \quad t \in I$$

holds, where u_0 is a nonnegative constant. Then

$$u(t) \leq u_0 e^{\int_0^t f(s)ds}, \quad t \in I.$$