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A novel image enhancement method using fuzzy Sure entropy

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ABSTRACT

Image enhancement is a very significant issue in image processing and analysis. In practice, many images (e.g.images captured from X-ray systems) are of low quality, such a slow-luminance and low-contrast, which must be enhanced before further processing. Fuzzy set theory is a useful tool for handling the ambiguity or uncertainty. Many researchers use the maximum Shannon entropy and fuzzy complement for image enhancement. But these methods are easy to be over-enhanced or under-enhanced or time-consuming. In this paper, a flexible method is proposed, which utilizes the maximum fuzzy Sure entropy, fuzzy c-partition and fuzzy complement (MSRM). Furthermore, a positive threshold value selection algorithm is developed to tune the enhancement performance of the proposed method. A variety of highly degraded images have been experimented by the proposed method. The comparisons of those experimental results show that the performance of our method overwhelms those of the existing ones.

1. Introduction

In practice, the quality of images is easy to be effected by several factors, such as the shooting angle, the shooting condition and the capturing approaches. Generally, the original images, which are captured by charging coupled device image sensors or by other digital image sensors in non-uniform illumination conditions, are unclear or blurred. Thus, the image without enhancement is impossible to exhibit the vivid details to observers directly. For instance, the X-rays images from direct digital radiography(DDR) system are not only blurred by noise, but also captured in non-uniform and low illumination conditions. So image enhancement is essential to these digital images and thus is an important task in image processing. The fuzzy set theory is a powerful tool for developing new and robust techniques in image processing [1–7]. A number of researchers have aimed for enhancing the low-contrasted image and many developed methods which perform quite well [8–10,13,19,20]. However, when applying on the low-luminance and low-contrasted images, those existing methods presented in the previous literatures cannot work well and most of the time lack compatibility and flexibility. For these reasons, we need to look for a suitable and flexible function to modify the intensity distribution of the image so as to fitting to human eyes. In this paper, a new method for image enhancement is proposed, which is based on the maximum fuzzy Sure entropy (MSRM). MSRM uses the fuzzy set theory [11], the fuzzy c-partition, the involutive fuzzy complements and the maximum fuzzy Sure entropy. We develop a new class of membership functions as well as a new measure of fuzziness. In our study, we design a new method to select a suitable positive threshold value to control the enhancement performance. To date, the Sure entropy principle has been rarely used in the literature [12]. We tested the proposed method and other existing four methods using various images of different types. The experimental results show that the proposed method achieves better performance in image enhancement, especially when the images are extremely low contrasted and low illuminated.

The rest of this paper is organized as follows. Section 2 describes an image in the form of fuzzy set theory, as well as the maximum fuzzy-Sure entropy method, fuzzy c-partition and the involutive membership functions. In Section 3, we describe the proposed method and the threshold value selection in full detail. The family of functions to modify the membership values of the gray levels is also introduced in this section. Section 4 presents the experimental results obtained using the proposed method and other existing methods. Comparisons and discussions of the proposed method and the other four contrast enhancement techniques are carried out. Finally, conclusions are made in Section 5.

2. Background

For image enhancement, we present the framework concerning fuzzy set theory, fuzzy entropy, fuzzy c-partition and involutive membership functions. Following, we describe the details.

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2.1. Image description in fuzzy set theory

In this paper, an image A of size M × N pixels, having L gray-levels ranging from $L_{\min}$ to $L_{\max}$, can be viewed as an array of fuzzy singletons [10,11]. Each element in the array is the membership value representing the degree of brightness of the gray level $g(g \in [L_{min}, L_{max}])$. In fuzzy set theory, image A can be written as below:

$$A = \{\mu_A(g_{ij})/g_{ij}, i = 1, 2, \cdots, M, j = 1, 2, \cdots, N\} \tag{1}$$

where $\mu_A(g_{ij})$ denotes the degree of brightness possessed by the gray level intensity g_{ij} corresponding to the (i, j) pixel. The histogram of the image is described as $h_A(g)$, $(g \in [L_{min}, L_{max}])$and denotes the frequency of occurrence of the gray level g. We introduce a membership function $P_A(g)$ of fuzzy set [7], which is the probability measure of the occurrence of gray-levels and described as $P_A(g) = \mu_A(g)\bullet \tilde{h}_A(g)$. Here $\tilde{h}_A(g)$denotes the probability of the gray level g by normalizing histogram $h_A(g)$. In our study, we write $\tilde{h}_A(g)$as below:

$$\tilde{h}_A(g) = \frac{h_A(g)}{M \times N} \tag{2}$$

So the probability of this fuzzy event can be calculated by:

$$P(A) = \sum_{g=0}^{L-1} \mu_A(g)\tilde{h}_A(g) \tag{3}$$

2.2. Fuzzy c-partition

The fuzzy c-partitions can be represented by partition matrices. It is defined as [13]. Let $T = \{t_1, t_2, \cdots, t_n\}$, Q_{cn} is a set of real c × n matrices, and c is an integer, $2 \le c \le n$. Fuzzy c-Partition space for T is the set:

$$M_C = \left\{ U \in Q_{cn} \middle| \mu_{ik} \in [0, 1];\ \sum_{i=1}^{c} \mu_{ik} = 1\ \forall\ k;\ 0 < \sum_{k=1}^{n} \mu_{ik} < n\ \forall\ i \right\} \tag{4}$$

2.3. Maximum entropy of fuzzy c-partition principle

In this section, we discuss two different measures of fuzziness for fuzzy c-partition, i.e. Shannon entropy and Sure entropy. The Shannon entropy principle was used for the maximum fuzzy entropy and fuzzy c-partition in early studies [14]. Let $U = \{A_1, A_2, \cdots, A_n\}$ be a finite partition of fuzzy sets. The Shannon entropy $H(U)$ [15] is defined as below:

$$H(\mathrm{U}) = -\sum_{i=0}^{c} P(A_i)\log P(A_i) \tag{5}$$

In this paper, we have investigated image enhancement performance of Sure entropy principle according to the Shannon entropy's for the maximum fuzzy entropy and fuzzy c-partition. The Sure entropy $H(U)$ presented in [16] is given as below:

$$|P(A_i)| \le \varepsilon \Rightarrow H(U) = \sum_{i=0}^{c} \min(P(A_i)^2, \varepsilon^2) \tag{6}$$

where $\varepsilon$is a positive threshold value. At the same time, since $P(A_i) \ge 0$, thus the Sure entropy $H(U)$ can be described as follows:

$$|P(A_i)| \le \varepsilon \Rightarrow H(U) = \sum_{i=0}^{c} \min(P(A_i), \varepsilon) \tag{7}$$

In this, we can tune the enhancement performance of the image by changing the value of ε.

2.4. Involutive membership functions

Image enhancement plays a key role in digital image processing, and there are a lots of literatures concerning this topic. When the image is improved, The gray-levels of the image histogram will be modified in some respects, e.g. by histogram equalization or appropriate gray-level transformation. In a word, the selection of a suitable function for the gray-level modification is an important step. In this study, we introduce Sugeno class of involutive fuzzy complements and present an involutive memberships [2,17]. Here, the membership values $\mu_A(g)(g \in [L_{min}, L_{max}])$ of image A denote the degree of compatibility of the gray level g with a relational image property (e.g. brightness, edginess etc.). Then we can define the involutive fuzzy complements as follows.

Definition 1. Let $\mu \in F(x)$ and $\alpha \in (0, 1)$. Then the complement of μ is the fuzzy set μ^*defined for all $g \in X$ by the membership function [17]:

$$\mu_A^*(g) = \frac{1 - \mu_A(g)}{1 + \lambda\mu_A(g)} \tag{8}$$

where

$$\lambda = \frac{1 - 2\alpha}{\alpha^2} \tag{9}$$

Properties of μ^*:

1. $\lambda \in (-1, \infty)$, so μ^*belongs to Sugeno's class of involutive complements.
2. $\mu_A^*(g) = \mu_A(g)$ if and only if $\mu_A(g) = \alpha$. Therefore, $\alpha$is the equilibrium of μ^*.
3. For $\alpha = 0.5$ the complement becomes the standard fuzzy complement, i.e., $\mu_A^*(g) = 1 - \mu_A(g)$.
4. Let $\mu_A, \mu_B \in F(X)$, and μ_A is α-sharper than μ_B. Then $\mu_A^*$is α-sharper than μ_B^*.
   In this study, for the image A, the membership function $\mu_A(ij)$ is initialized as follows:

$$\mu_A(ij) = \frac{L_{ij} - L_{\min}}{L_{\max} - L_{\min}} \tag{10}$$

where $L_{ij} \in [L_{min}, L_{max}]$,$i = 1, 2, \cdots, M$, and $j = 1, 2, \cdots, N$. Referring to the literature [17], we have new involutive memberships as below:

$$\hat{\mu}_A(ij) = \begin{cases} \frac{1}{\alpha}\mu_A(ij)^2, & \text{if} \quad \mu_A(ij) \in [0, \alpha] \\ \frac{1 - B(\mu_A(ij))}{1 + \lambda B(\mu_A(ij))}, & \text{if} \quad \mu_A(ij) \in [\alpha, 1] \end{cases} \tag{11}$$

where

$$B(\mu_A(ij)) = \frac{1}{\alpha}\left(\frac{1 - \mu_A(ij)}{1 + \lambda\mu_A(ij)}\right)^2 \tag{12}$$

The relationship of α and λ is depicted in (9), we can calculate λ and replace it into (12) and (11). Then, we obtain the involutive memberships $\mu_A(ij)$, called α-involutive memberships as depicted in Fig. 1.

The α-involutive fuzzy class begins with a high-contrast image and by increasing α the image changes itself into a dim image. In this paper, we obtain the optimal parameter α using the exhausted search method.

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