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Some interesting properties of the fuzzy linguistic model based on discrete fuzzy numbers to manage hesitant fuzzy linguistic information

Juan Vicente Riera [a], Sebastià Massanet [a,*], Enrique Herrera-Viedma [b], Joan Torrens [a]

[a] Department of Mathematics and Computer Science, University of the Balearic Islands, Ctra. de Valldemossa, Km.7.5, 07122 Palma de Mallorca, Spain
[b] Department of Computer Science and Artificial Intelligence, University of Granada, Granada, Spain

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ABSTRACT

The management of hesitant fuzzy information is a topic of special interest in fuzzy decision making. In this paper, we focus on the use and properties of the fuzzy linguistic modelling based on discrete fuzzy numbers to manage hesitant fuzzy linguistic information. Among these properties, we can highlight the existence of aggregation functions with no need of transformations or the possibility of a greater flexibilization of the opinions of the experts, even using different linguistic chains (multigranularity). Furthermore, based on these properties we perform a comparison between this model and the one based on hesitant fuzzy linguistic term sets, showing the advantages of the former with respect to the latter. Finally, a fuzzy decision making model based on discrete fuzzy numbers is proposed.

1. Introduction

The presence of uncertainty is a common feature that characterizes a wide range of real problems related to decision making. Thus, for instance, a group of experts should decide the feasibility of a financial transaction from information which is often vague or incomplete, or similarly, when a medical team must make a diagnosis based on preliminary tests. Frequently, the uncertainty appears when we use assessments not necessarily quantitative but we deal with terms or qualitative information when making the decision. For this reason, the fuzzy linguistic approximations have emerged as a tool which allow to properly handle the qualitative information. In this sense, we highlight the *symbolic linguistic model* based on ordinal scales (wherein an order is considered among the different linguistic labels) [11,14]; *the linguistic 2-tuples model* [15], which introduces the symbolic translation to the linguistic representation; *the linguistic model based on type-2 fuzzy sets representation* [32], which represents the semantics of the linguistic terms using type-2 fuzzy membership functions; *the proportional 2-tuple model* [34], which extends the 2-tuple model by using two linguistic terms with their proportion to model the information, or *the linguistic model based on PSO and granular computing of linguistic information* [2], which proposes to model the linguistic information like expressed in terms of information granules defined as sets, among many others. A common feature of many of these models is that experts should express their valuations choosing a single linguistic level associated with the linguistic variable. This kind of information is usually interpreted using a linguistic scale like this,

$$\mathcal{L} = \{\text{EB}, \text{VB}, \text{B}, \text{F}, \text{G}, \text{VG}, \text{EG}\} \quad (1)$$

where the linguistic terms correspond to the expressions *"Extremely Bad"*, *"Very Bad"*, *"Bad"*, *"Fair"*, *"Good"*, *"Very Good"* and *"Extremely Good"* respectively. However in many cases the experts' opinions do not correspond exactly to a particular linguistic term. On the contrary, expressions like *"better than good"* or *"between fair and very good"* or, even more complex ones, are commonly used by experts in order to make their opinions. Recently, V. Torra [31] introduced the hesitant fuzzy sets as a possible generalization of the classical fuzzy sets. Based on this idea, different authors [29,30,36,37] have proposed a computational linguistic model based on hesitant fuzzy linguistic term sets, allowing that the expert could consider several possible linguistic values or richer expressions than a single term for an indicator, alternative, variable, etc; increasing the richness of linguistic elicitation based on fuzzy linguistic approach. Several methods based on hesitant fuzzy

* Corresponding author. Tel.: +34 971259915.
E-mail addresses: jvicente.riera@uib.es (J.V. Riera), s.massanet@uib.es (S. Massanet), viedma@decsai.ugr.es (E. Herrera-Viedma), jts224@uib.es (J. Torrens).

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linguistic term sets for group decision-making have been recently proposed (see [6,17,18,35,40]). On the other hand, the authors [4,5,21,26–28] have considered another linguistic computational model based on discrete fuzzy numbers [33], which allows to interpret the qualitative information in a more flexible way and also enables to aggregate the information they expressed using this type of fuzzy subsets directly. Although at a first glance the two models seems to be different, they are in fact quite connected. As we will prove, the hesitant fuzzy linguistic term sets used by the first model can be interpreted as particular cases of discrete fuzzy numbers and although the aggregation functions used in the aggregation phase presented in the original articles (see [29] for the hesitant fuzzy sets based one and [21] for the discrete fuzzy numbers based one) are different (as it is the exploitation phase), if both models used the adequate aggregation functions and the same exploitation method, the results would coincide.

In this article we want to make a step further and the main goal will be to show the advantages of using the model based on discrete fuzzy numbers with respect to the hesitant fuzzy sets based one. These advantages are related to the existence of aggregation functions on the set of discrete fuzzy numbers which allow us to aggregate the opinions without any transformation or loss of information, to the possibility of a greater flexibilization of the opinions given by the experts and finally, to the possibility of the experts to evaluate using different linguistic chains (multigranularity [23]). We will also present an example of a multi-criteria decision making problem using the linguistic model based on discrete fuzzy numbers, showing the commented advantages of this method.

The paper is organized as follows. In Section 2, we make a brief review of discrete fuzzy numbers and hesitant fuzzy linguistic term sets. In Section 3, we explain the main characteristics of the fuzzy linguistic model based on discrete fuzzy numbers, comparing this method with the model based on hesitant fuzzy linguist term sets. Throughout the process, we prove that the discrete fuzzy numbers allow a greater flexibilization and an easier management of the opinions given by the experts. In Section 4, we propose a multi-criteria decision making approach based on these particular fuzzy subsets using extensions of two well known discrete parametric compensatory aggregation functions [8,20]. The last section is devoted to give some conclusions and the future work that we want to develop.

2. Preliminaries

In this section, we recall some definitions and results about aggregation functions and discrete fuzzy numbers which will be used later. We also recall the notion of hesitant fuzzy linguistic term sets and some operations related to this concept.

2.1. Aggregation functions on bounded partially ordered sets

Let $(P; \leq)$ be a non-trivial bounded partially ordered set (poset) with 0 and 1 as minimum and maximum elements respectively.

Definition 1. An n-ary aggregation function on P is a function $F: P^n \to P$ such that it is increasing in each component, $F(0, \ldots, 0) = 0$ and $F(1, \ldots, 1) = 1$.

Remark 1. Of course, the number of inputs to be aggregated can be different in each case. Thus, aggregation functions are commonly defined not on P^n, but on $\cup_{n\geq 1} P^n$ and then they are usually called *extended aggregation functions*. An easy way to construct extended aggregation functions is from associative binary aggregation functions.

An important case is when we take as poset a finite chain L_n with $n+1$ elements. In such a framework, only the number of elements is relevant (see [24]) and so the simplest finite chain, that is $L_n = \{0, 1, \cdots, n\}$, is usually considered. On the other hand, it is well known that qualitative information is often interpreted as values in a totally ordered finite scale $\mathcal{L}$ like this:

$$\mathcal{L} = \{\mathrm{EB}, \mathrm{VB}, \mathrm{B}, \mathrm{F}, \mathrm{G}, \mathrm{VG}, \mathrm{EG}\}. \tag{2}$$

In these cases, the representative finite chain L_n is used to model these linguistic hedges (L_6 for the previous scale (2)) and several researchers have developed an extensive study of aggregation functions on L_n, usually called *discrete aggregation functions* [9,10,16,19,20,24,39].

2.2. Discrete fuzzy numbers

By a fuzzy subset of $\mathbb{R}$, we mean a function $A : \mathbb{R} \to [0, 1]$. For each fuzzy subset A, let $A^\alpha = \{x \in \mathbb{R} : A(x) \geq \alpha\}$ for any $\alpha \in (0, 1]$ be its α-level set (or α-cut). By $supp(A)$, we mean the support of A, i.e. the set $\{x \in \mathbb{R} : A(x) > 0\}$. By A^0, we mean the closure of $supp(A)$.

Definition 2. [33] A fuzzy subset A of $\mathbb{R}$ with membership mapping $A : \mathbb{R} \to [0, 1]$ is called a *discrete fuzzy number* if its support is finite, i.e., there exist $x_1, \ldots, x_n \in \mathbb{R}$ with $x_1 < x_2 < \ldots < x_n$ such that $supp(A) = \{x_1, \ldots, x_n\}$, and there are natural numbers s, t with $1 \leq s \leq t \leq n$ such that:

1. $A(x_i)=1$ for any natural number i with $s \leq i \leq t$ (*core*)
2. $A(x_i) \leq A(x_j)$ for each natural number i, j with $1 \leq i \leq j \leq s$.
3. $A(x_i) \geq A(x_j)$ for each natural number i, j with $t \leq i \leq j \leq n$.

Remark 2. If the fuzzy subset A is a discrete fuzzy number then the support of A coincides with its closure, i.e. $supp(A) = A^0$.

From now on, we will denote the set of discrete fuzzy numbers using the abbreviation *DFN* and *dfn* will denote a discrete fuzzy number. Also, we will denote by $\mathcal{A}_1^{L_n}$ the set of all discrete fuzzy numbers whose support is a subinterval of the finite chain L_n. Moreover, note that for any $A \in \mathcal{A}_1^{L_n}$, not only $supp(A)$ is an interval of L_n, but also any α-level set. So, let $A, B \in \mathcal{A}_1^{L_n}$ be two discrete fuzzy numbers, and we will denote by $A^\alpha = [x_1^\alpha, x_p^\alpha]$, $B^\alpha = [y_1^\alpha, y_k^\alpha]$ the α-level cuts for A and B, respectively.

The authors showed in [3] that $\mathcal{A}_1^{L_n}$ is a bounded distributive lattice while the set of discrete fuzzy numbers in general is not.

Aggregation functions defined on L_n have been extended to the bounded lattice $\mathcal{A}_1^{L_n}$ (see for instance [4,26]) according to the next result.

Theorem 1. [4,26] *Let consider a binary aggregation function F on the finite chain L_n. The binary operation on $\mathcal{A}_1^{L_n}$ defined as follows*

$$\mathcal{F} : \mathcal{A}_1^{L_n} \times \mathcal{A}_1^{L_n} \longrightarrow \mathcal{A}_1^{L_n}$$
$$(A, B) \longmapsto \mathcal{F}(A, B)$$

being $\mathcal{F}(A, B)$ the discrete fuzzy number whose α-cuts are the sets

$$\{z \in L_n \mid \quad \min F(A^\alpha, B^\alpha) \leq z \leq \max F(A^\alpha, B^\alpha)\}$$

for each $\alpha \in [0, 1]$ is an aggregation function on $\mathcal{A}_1^{L_n}$. This function will be called the extension of the discrete aggregation function F to $\mathcal{A}_1^{L_n}$. In particular, if F is a t-norm, a t-conorm, a uninorm or a nullnorm, its extension $\mathcal{F}$, so is. In addition, if F is a compensatory aggregation function in L_n, so is its extension $\mathcal{F}$.

Example 1. Let us consider the finite chain $L_8 = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ and $A = \{0.3/0, 0.5/1, 1/2, 0.3/3\}$, $B = \{0.3/2, 0.5/3, 1/4, 0.8/5\} \in \mathcal{A}_1^{L_8}$.