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Recursive state estimation for complex networks with random coupling strength ☆

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A B S T R A C T

This paper studies the state estimation problem for complex networks with random coupling strength. Unlike the constant coupling strength used in the existing models, the coupling strength is assumed to be chosen from a uniform random distribution with non-negative mean. By employing the structure of the extended Kalman filter (EKF), a recursive state estimator is developed where the gain matrix is determined by optimizing an upper bound matrix despite the random coupling terms and linearization errors. Compared with the augmented approach for state estimation of complex networks, an important feature of the proposed estimator is that the gain matrix can be derived for each node separately. By using the stochastic analysis techniques, sufficient conditions are established to guarantee that the estimation error is bounded in mean square. Simulation results are provided to show the effectiveness and applicability of the proposed estimator.

1. Introduction

Complex networks have received great attention in recent years due to their potential applications in many real-world systems such as sciences, engineering and society [1–12]. In particular, special attention has been paid to the state estimation problem for complex networks because it helps understand the intrinsic structure of the networks. Compared with the state estimation for an isolated node, the problem of state estimation for complex networks becomes more challenge due to the coupling features between nodes.

To overcome the difficulty of the coupling features in the state estimation of complex networks, many strategies have been proposed with respect to different network issues. These issues include, but are not limited to, missing measurements, time delays, and randomly occurred nonlinearities. For example, an estimator has been developed for uncertain complex networks with missing measurements and time delays in [13]. In [14], a bounded H_∞ state estimator has been proposed for complex networks over a finite-time horizon. The H_∞ estimators have been also designed for complex networks with uncertain coupling strength and incomplete measurements [15,16]. The robust state estimation problem has been studied for uncertain complex networks with time-varying delays [17] and an integrated approach has been proposed to address the problem of global synchronization and state estimation for nonlinear singularly perturbed complex networks [18]. Event-triggered state estimation for complex networks with mixed time-delays has been studied in [19] and the results have been extended by using the Round-Robin protocol [20]. A finite-time estimator has been proposed for complex networks with jump Markov parameters in [21] and an asynchronous dissipative estimator has been proposed for complex networks with quantized jumping coupling and uncertain measurements in [22]. It should be pointed out that almost all the aforementioned work focus on developing time-invariant gain matrices by solving linear matrix inequalities. Recently, a recursive state estimator has been proposed for stochastic coupling networks with missing measurements [23], where the time-variant gain matrix is determined at each time instant. An important feature of the estimator in [23] is that all the states of the nodes are formulated into an augmented vector and all the gain matrices for the nodes are determined simultaneously according to the augmented vector. Thus, the computational cost might be very high for complex networks with a large number of nodes and the estimation algorithms cannot be implemented in a distributed fashion. This motivates us to develop a non-augmented approach for state estimation of complex networks. Moreover, the stability analysis of

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the estimator in [23] is not provided. On the other hand, most of studies assume that the coupling strength is constant in the complex networks. Although the synchronization problem for complex networks with random coupling strength has been studied recently [24–26], the state estimation for complex networks with random coupling strength remains an open challenging issue.

In this paper, we attempt to develop a novel state estimator for complex networks with random coupling strength. Inspired by the Kuramoto model with randomly coupled oscillators in [27] where the coupling strength is chosen from a uniform random distribution in the domain $[-1, 1]$, the coupling strength is assumed to be chosen from a uniform random distribution with non-negative mean in this paper. Then, the structure of the extended Kalman filter (EKF) is adopted to develop a recursive estimator by proposing the predicted and updated estimation error systems. Due to the linearization errors and the random coupling strength, the variance-constrained approach is adopted to pursuing upper bound matrices for the predicted and the updated covariance matrices so that the gain matrix can be derived for each node separately by minimizing the trace of the upper bound matrix. By using the stochastic analysis techniques, it is shown that the estimation error is bounded in mean square under certain conditions. Finally, a numerical example involving tracking four interacting mobile robots is provided to verify the effectiveness of the proposed estimator. The main contributions can be summarized as follows: (1) Compared with the complex networks with deterministic coupling strength, the model considered in this paper is more comprehensive. Moreover, the stability analysis has been provided for the proposed estimator whereas it has not been studied even for the complex networks with deterministic coupling strength. (2) The gain matrices of the proposed estimator can be derived separately for each node, whereas they are derived simultaneously for all nodes. Thus, the proposed estimator facilitates developing distributed algorithms.

The remainder of this paper is organized as follows. The state estimation problem is formulated in Section 2. The upper bound matrices for the state estimation error covariances are derived to design the gain matrices and the stability analysis is provided in Section 3. Simulation results are provided to verify the effectiveness of the proposed estimator in Section 4. Conclusions are drawn in Section 5.

2. Problem statement

In this paper, we consider the following complex network with random coupling strength

$$x_{i,k+1} = f(x_{i,k}) + \sum_{j=1}^{N} \omega_{ij,k} \Gamma x_{j,k} + w_{i,k} \tag{1}$$

$$z_{i,k} = H_{i,k} x_{i,k} + v_{i,k} \tag{2}$$

where $x_{i,k} \in \mathbb{R}^n$ and $z_{i,k} \in \mathbb{R}^p$ denote the state vector and the measurement vector, respectively. i and k denote the node index and the time instant, respectively. $f(\cdot)$ is a known nonlinear function that is assumed to be continuously differentiable. $H_{i,k}$ is the measurement matrix for the i-th node. Γ is the inner-coupling matrix. $\omega_{ij,k}$ is the coupling strength which is assumed to be chosen from a uniform random distribution in the domain $[a_i, b_i]$, i.e., $\omega_{ij,k} \sim \mathcal{U}(a_i, b_i)$. The mean and the variance of the random variable $\omega_{ij,k}$ are taken to be $\lambda_i \geq 0$ and $\sigma_i \geq 0$, respectively. The process noise $w_{i,k}$ and the measurement noise $v_{i,k}$ are zero-mean white Gaussian with covariance matrices $Q_{i,k}$ and $R_{i,k}$, respectively. It is assumed that the random variables $w_{i,k}$, $v_{i,k}$ and $\omega_{ij,k}$ are mutually uncorrelated for any i, j, k.

To derive the state estimate for each node, the structure of the EKF is adopted to develop a recursive state estimator

$$\bar{x}_{i,k} = f(\hat{x}_{i,k}) + \sum_{j=1}^{N} \lambda_i \Gamma \hat{x}_{j,k} \tag{3}$$

$$\hat{x}_{i,k+1} = \bar{x}_{i,k} + K_{i,k+1}(z_{i,k+1} - H_{i,k+1}\bar{x}_{i,k}) \tag{4}$$

where $\bar{x}_{i,k}$ and $\hat{x}_{i,k+1}$ denote the predicted and the updated estimates at time instant $k+1$, respectively. $K_{i,k+1}$ is the gain matrix to be determined.

As in the EKF, the updated estimation error and the corresponding covariance are defined as follows:

$$e_{i,k+1} = x_{k+1} - \hat{x}_{i,k+1} \tag{5}$$

$$P_{i,k+1} = \mathbb{E}\{e_{i,k+1} e_{i,k+1}^T\} \tag{6}$$

The aim of this paper is to design filters described by (3) and (4), such that there exists a sequence of positive-definite matrices $\Phi_{i,k+1}$ satisfying

$$P_{i,k+1} \leq \Phi_{i,k+1} \tag{7}$$

The gain matrix $K_{i,k+1}$ is determined by minimizing the trace of the upper bound matrix $\Phi_{i,k+1}$ at each time instant.

Remark 1. Compared with the existing coupling network models in the literature, the system model investigated in the paper is comprehensive. The coupling strength is always assumed to be positive constant in the existing models [15–17,23], while it is assumed to be an uniformly distributed random variable in this paper. Notice that the coupling strength is not restricted be positive at different time instants in this paper.

Remark 2. In [23], a recursive state estimator has been developed for complex networks with missing measurements. Although the problem setup of this paper seems close to [23], the determination of the gain matrix is substantially different. In [23], all the estimation errors are formulated in an augmented vector and an overall upper bound matrix is proposed for the augmented estimation error covariance. Thus, the gain matrices should be determined simultaneously for all nodes by minimizing the trace of the overall upper bound matrix. A notable feature of the proposed filter in this paper is that an upper bound matrix is provided for each node's estimation error covariance so that the gain matrix can be determined separately for each node.

3. Main results

In this section, an upper bound matrix is derived for the updated estimation error covariance and the gain matrix is determined with respect to the upper bound matrix. The stability analysis is also presented for the proposed estimator under certain conditions.

3.1. The proposed estimator

To derive an upper bound matrix for the updated estimation error covariance, the following lemmas are important.

Lemma 1 ([28]). *Given matrices A, B, C and D with appropriate dimensions such that $CC^T \leq I$. Let U be a symmetric positive definite matrix and $a > 0$ be an arbitrary positive constant such that $a^{-1}I - DUD^T > 0$. Then the following matrix inequality holds*

$$(A + BCD)U(A + BCD)^T \leq A(U^{-1} - aD^TD)^{-1}A^T + a^{-1}BB^T \tag{8}$$

Lemma 2 ([29]). *For $0 \leq k < n$, suppose that $A = A^T > 0$. Let $\varphi_k(\cdot)$ and $\psi_k(\cdot)$ be two sequences of matrix functions such that*