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Adaptive synchronization of delayed reaction-diffusion neural networks with unknown non-identical time-varying coupling strengths

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ABSTRACT

This paper concerns the asymptotic synchronization of delayed reaction-diffusion neural networks (RDNNs) with unknown nonidentical time-varying coupling strengths, where the time-varying coupling strengths are consist of continuous time-varying periodic parameters and time-invariant nonnegative parameters. By utilizing a novel adaptive approach, the differential-difference type adaptive laws of coupling strengths and adaptive controller are designed such that the nonidentical RDNNs are asymptotic synchronization. The sufficient conditions dependent on the reaction-diffusion terms are derived by constructing a novel Lyapunov-Krasovskii-like composite energy functional (CEF) and using Barbalat's lemma. Finally, a simulation example is provided to illustrate the effectiveness of the developed approach.

1. Introduction

In the past few decades, the analysis of neural networks (NNs) has received a great deal of attention due to their potential applications in many areas, such as optimization solver, image processing, pattern recognition, and quadratic programming problems [1–4]. In real world situation, the existence of time delay may lead to instability and/or deteriorate the performance of the underlying neural networks [4]. As a result, the synchronization of NNs with delays has been studied in an extensive literature [5–13].

In artificial NNs, strictly speaking, diffusion effects cannot be avoided when electrons are moving in asymmetric electromagnetic fields [14–24]. Thus, it is significant to investigate the neural networks with diffusion terms which can be described by partial differential equations. The study on synchronization problem for RDNNs can be seen in [19–24], and references therein. In [19], using the drive-response concept, a state feedback control law was designed to study the asymptotic synchronization for a class of RDNNs with time-varying delays. In [21], a synchronization controller was designed by using adaptive feedback control techniques. Then, based on the inequality techniques and Lyapunov method, several criteria were proposed for adaptive synchronization and parameters identification of uncertain delayed neural networks. The synchronization problem of generalized stochastic RDNNs with mixed time-varying delays was investigated by using linear feedback control in [24], and Lyapunov stability theory combining with stochastic analysis approaches was employed to derive sufficient criteria of the stochastic synchronization.

The research of neural networks involves not only the dynamic analysis of equilibrium point but also that of periodic oscillatory solution. As is well known, an equilibrium point can be viewed as a special case of periodic solution with an arbitrary period or zero amplitude [25]. In this sense, the analysis of periodic solutions of neural networks may be considered to be more general than that of equilibrium points. The related results on the analysis of periodic solutions of NNs can see [16,25–28], and references therein.

In the existing literature, the coupling strengths in the reaction-diffusion neural networks are assumed to be either known constants or unknown constants. There are seldom results on the coupling strengths being unknown time-varying in the reaction-

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diffusion neural networks. It is well-known that the synthesis and control of the systems with unknown time-varying parameters is a longstanding open problem. Recently, the period adaptive controllers were proposed for the unknown time-varying parametric systems with known periodicity in [29–31], then the method were used to solve the synchronization problems for complex dynamical networks with unknown time-varying coupling strengths [32–37]. More recently, the synchronization of the reaction-diffusion neural networks with unknown time-varying coupling strengths was presented in [38]. It should be pointed out that, the problem addressed in [38] is the synchronization of two identical delayed RDNNs with different initial condition, since the coupling strengths and parameters of the drive and response systems are the same. From the point of view of engineering, it is very difficult to keep the two NNs to be identical in all the time [5,6]. Therefore, the study on synchronization of nonidentical RDNNs is more essential and useful in real-world applications [6]. Up till now, there are some studies on the synchronization for nonidentical NNs [5,6]. However, there are few results on the synchronization of RDNNs with nonidentical unknown time-varying coupling strengths. This motivates the present study.

In this paper, we extend the synchronization strategy of delayed RDNNs with unknown identical time-varying coupling strengths [38] to the case of that with unknown nonidentical time-varying coupling strengths. The coupling strengths in the driven network and the response network are consists of different unknown continuous time-varying periodic parameters and unknown time-invariant nonnegative parameters. Thus, the problem becomes more complex and challenging. The main contribution of the paper is that the continuous-discrete type adaptive laws of coupling strengths and adaptive controller are proposed by a new adaptive methodology, which make the nonidentical reaction-diffusion neural networks be asymptotic synchronization. The sufficient condition is derived in terms of simple inequalities by constructing a novel Lyapunov-Krasovskii functional, which is quite different from the condition in [38] and easy to verify.

The rest of this paper is organized as follows: Section 2 presents the model of RDNNs and some preliminaries. The adaptive controller and adaptive laws of coupling strengths are designed in Section 3. The asymptotic synchronization criteria are proposed in Section 4. Section 5 provides an illustrative example to show the effectiveness of the proposed approach. Section 6 concludes this paper.

Notations: R^n denote the n dimensional Euclidean space. $\Omega = \left\{ x = (x_1, x_2, \ldots, x_m)^T \middle| |x_k| \le l_k, k = 1, 2, \ldots, m \right\}$is a bounded compact set with smooth boundary $\partial\Omega$ and $mes\Omega > 0$ in space R^m, and l_k is a constant. $C\left(\left[-\tau, 0\right] \times \Omega; R^n\right)$ denotes the Banach space of continuous functions which maps $\left[-\tau, 0\right] \times \Omega$ into R^n with the topology of uniform converge. $L^2(\Omega)$ is the space of real functions on Ω which are L^2 for the Lebesgue measure. It is a Banach space for the norm$\|u(t, x)\|_2 = \left(\sum_{i=1}^n \|u_i(t, x)\|_2^2\right)^{1/2}$, with $u(t, x) = (u_1(t, x), u_2(t, x), \ldots, u_n(t, x))^T$ and $\|u_i(t, x)\|_2 = \left(\int_\Omega |u_i(t, x)|^2 dx\right)^{1/2}$. For any $\phi(t, x) \in C([-\tau, 0] \times \Omega; R^n)$, we define $\|\phi(t, x)\|_2 = \sup_{-\tau \le s \le 0} \left(\sum_{i=1}^n \|\phi_i(s, x)\|_2\right)^{1/2}$, where $\|\phi_i(s, x)\|_2 =\left(\int_\Omega \left|\phi_i(s, x)\right| dx\right)^{1/2}$.

2. Networks model and preliminaries

The delayed RDNNs with unknown time-varying coupling strengths is described by

$$\frac{\partial u_i(t, x)}{\partial t} = \sum_{k=1}^m \frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial u_i(t, x)}{\partial x_k}\right) - \eta_{1i}(t, x)a_i u_i(t, x) + \beta_{1i}(t, x)\sum_{j=1}^n b_{ij} f_j(u_j(t, x)) + \varphi_{1i}(t, x) \sum_{j=1}^n c_{ij} f_j(u_j(t - \tau_j(t), x)) + J_i, \quad i = 1, 2, \ldots, n \tag{1}$$

where n is the number of neurons in the networks; $x=(x_1, x_2, \ldots, x_m)^T \in \Omega$; $u_i(t, x)$ is the state of ith neuron at time t and in space x; $D_{ik} = D_{ik}(t, x) > 0$ denotes the transmission diffusion coefficient along the ith neuron; $f_j(\cdot)$ denotes the activation function of jth neuron; $\eta_{1i}(t, x)$, $\beta_{1i}(t, x)$ and $\varphi_{1i}(t, x)$ are the unknown time-varying coupling strengths for the ith neuron; $a_i > 0$ represents the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the networks and external inputs; b_{ij} and c_{ij} stand for the weights for neuron interconnections; $\tau_j(t)$ is the transmission delay and satisfies $0 < \tau_j(t) < \tau$ (τ is a constant); J_i denotes the external input on the ith neuron.

The boundary condition and initial condition associated with the RDNNs (1) are given by

$$u_i(t, x) = 0, \quad (t, x) \in [-\tau, +\infty) \times \partial\Omega \tag{2}$$

$$u_i(t, x) = \phi_i(t, x), \quad (t, x) \in [-\tau, 0] \times \Omega \tag{3}$$

where $\phi(t, x) = (\phi_1(t, x), \phi_2(t, x), \ldots, \phi_n(t, x))^T \in C([-\tau, 0] \times \Omega; R^n)$.

Let the delayed RDNNs (1) be the drive system, and the corresponding response system is given as

$$\frac{\partial \tilde{u}_i(t, x)}{\partial t} = \sum_{k=1}^m \frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial \tilde{u}_i(t, x)}{\partial x_k}\right) - \eta_{2i}(t, x)a_i\tilde{u}_i(t, x) + \beta_{2i}(t, x)\sum_{j=1}^n b_{ij} f_j(\tilde{u}_j(t, x)) + \varphi_{2i}(t, x)\sum_{j=1}^n c_{ij} f_j(\tilde{u}_j(t - \tau_j(t), x)) + J_i + v_i(t, x), \quad i = 1, 2, \ldots, n \tag{4}$$

where $\tilde{u}_i(t, x)$ is the state of ith neuron of the response system; $v_i(t, x)$ is the control input to be designed; $\eta_{2i}(t, x)$, $\beta_{2i}(t, x)$ and $\varphi_{2i}(t, x)$ are the unknown time-varying coupling strengths for the ith neuron.

The boundary condition and initial condition for system (4) are given as

$$\tilde{u}_i(t, x) = 0, \quad (t, x) \in [-\tau, +\infty) \times \partial\Omega,$$

$$\tilde{u}_i(t, x) = \tilde{\phi}_i(t, x), \quad (t, x) \in [-\tau, 0] \times \Omega,$$

where $\tilde{\phi}(t, x) = (\tilde{\phi}_1(t, x), \tilde{\phi}_2(t, x), \ldots, \tilde{\phi}_n(t, x))^T \in C([-\tau, 0] \times \Omega; R^n)$.

Define the synchronization error signal as $e_i(t, x) = \tilde{u}_i(t, x) - u_i(t, x)$ and $e(t, x) = (e_1(t, x), e_2(t, x), \ldots, e_n(t, x))^T$, then the error dynamical system can be obtained from (1) and (4) as follows: