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On lattice ordered soft sets

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ABSTRACT

Certain type of linguistic terms such as satisfactory, good, very good and excellent have an order among them. In this paper we introduce a new concept of soft sets with some order among the parameters. Some properties of lattice ordered soft sets are given. Lattice ordered soft sets are very useful in particular type of decision making problems where some order exists among the elements of parameters set.

1. Introduction

In daily life, one usually comes across with linguistic terms having certain order among them. That is, a ranking exist in these terms. For example to tell the quality of a certain product a sale person employee terms like unsatisfactory, satisfactory, good, very good, and excellent. Clearly these terms have an order among them. This type of ranking, to explain the quality of a product helps the customers to select a particular brand of a product. In mathematics partial ordered sets or lattices provide a very nice abstraction for such notions.

Fuzzy set theory [17] is a very nice tool to handle linguistic terms, particularly when there exists some order among them. But defining a membership grade for the elements of a set is a major difficulty in fuzzy set theory [15]. This difficulty may be due to lack of parameterization tools in fuzzy sets [15]. Therefore Molodtsov introduced the concept of soft sets. In soft sets theory there are enough number of parameters available to handle uncertainty, so that its free from difficulties associated with fuzzy sets. Another advantage of soft set theory is that, it can preserve crisp or fuzzy data in a very nice way. At present fuzzy sets, rough sets and soft sets are among major tools to deal with uncertainty. These theories are quite different in their nature yet authors try establish some linkage among them. For details see [1,4,8].

During recent years soft set theory has emerged as a new mathematical tool to deal with uncertainty. Due to its applications in diverse fields researchers, practitioners and mathematicians are taking keen interest in it. Now literature is quite rich on applications of soft sets ranging from algebra to decision making. For details see [2,5,7,10,11,13]

As mentioned above, sometimes linguistic terms have particular ranking among them. These terms can be considered as parameters, so the collection of such terms give rise to a parameters set. For example, in a class of an educational institute to establish a ranking among student's performance, we say good students of the class, it may mean those students who obtained over seventy percent marks. Similarly when we say very good or excellent students, it may mean those students who got more that eighty or ninety percent marks respectively. In this situation there is an order among the terms (parameters) good, very good and excellent. This situation gives rise to idea of an anti-lattice order soft set because number of students may decrease as percentage of obtained marks goes higher. According to our knowledge, in soft set theory up-till now, no study exists where an order has been considered among the elements of parameters set. Moreover order among the parameters may not be linear, therefore in the present paper we introduce the notion of lattice (anti-lattice) ordered soft sets. It is well known that a soft set is a map from the set of parameters to the power set of the universe set. If set of parameters is a lattice then a monotone or an isotone map give rise to the notions of lattice ordered soft sets and anti-lattice ordered soft sets respectively. Therefore it is imperative to study such soft sets, where some type of ranking exists among the elements of parameters set.

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This paper has been arranged as the following. In Section 2, some basic notions about lattices and soft set sets are given. These notions will be employed in the sequel. Concept of lattice (anti-lattice) ordered soft set is introduced in Section 3. In the same section some properties of these are studied. Algebraic structures of lattice (anti-lattice) ordered soft sets are investigated in Section 4. Lattice (anti-lattice) ordered soft set soft sets can be very helpful in decision making problems, where set of parameters have an order among its elements. An example is given in Section 5, to explain this idea. This example is verified by well known technique of TOPSIS.

2. Preliminaries

In this section some basic notions and results about lattices and soft sets are given. These terms will be required in the sequel. A binary relation $\leq$ defined on a non-empty set A is called a partial order on the set A if it is reflexive, antisymmetric and transitive. If in addition, for every $a,b \in A$, such that $a \neq b$, either $a \leq b$ or $b \leq a$, then we say $\leq$ is total order on A. A non-empty set with partial order on it is called a partially ordered set, or more briefly a poset. And if the relation is a total order then we speak it a totally ordered set or simply a chain.

A lattice L is a poset in which for all $a, b \in L$ the set $\{a, b\}$ has a supremum "$\vee$" and an infimum "$\wedge$". If there are elements 0 and 1 in L such that $0 \leq x$ and $x \leq 1$ for all $x \in L$. Then L is called a bounded lattice. A lattice in which either of the distributive laws hold is called distributive lattice. If De Morgan's laws hold for a bounded distributive lattice having an involution, then it is called a De Morgan's lattice or De Morgan's algebra. A De Morgan's algebra $(L, \wedge, \vee, ^c, 0, 1)$ that satisfies $x \wedge x^c \leq y \vee y^c$ for all $x, y \in L$, is called Kleene algebra.

Next some notions about soft sets are given.

Definition 1. [15] Let U be an initial universe, E be the set of all possible parameters under consideration with respect to U and A be a subset of E. Then a pair (F, A) is called a soft set over U, where F is a mapping $F : A \to \mathcal{P}(U)$.

Definition 2. [12] For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a *soft subset* of (G, B) if

1 $A \subseteq B$ and
2 $F(e) \subseteq G(e)$ for all $e \in A$.

We write $(F, A)\widetilde{\subset}(G, B)$. *In this case* (G, B) *is said to be a soft super set of* (F, A).

Definition 3. [12] Two soft sets (F, A) and (G, B) over a common universe U are said to be *soft equal* if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 4. For two soft sets $(F, A)$and (G, B) over a common universe U, we say that (F, A) is a *soft twisted subset* of (G, B) if

1 $A \subseteq B$ and
2 $G(a) \subseteq F(a)$ for all $a \in A$.

We write $(F, A)\widehat{\subset}(G, B)$. *In this case* (G, B) *is said to be a soft twisted super set of* (F, A). *If* $A = B$ *then* (F, A) *is a soft twisted subset of* (G, B) *if and only if* (G, B) *is a soft subset of* (F, A).

It should be noted that (F, A) *is soft equal to* (G, B) *if* (F, A) *is a soft twisted subset of* (G, B) *and* (G, B) *is a soft twisted subset of* (F, A).

Example 1. Let $U = \{h_1, h_2, h_3, h_4, h_5, h_6, h_7\}$ be a set. Let $A = \{e_1, e_2, e_3, e_4\}$, $B = \{e_1, e_2, e_3, e_4, e_5\}$ be sets of parameters. Consider the soft sets (F, A) and (G, B)such that $(F, A) = \{F(e_1) = \{h_1, h_2, h_3, h_4\}, F(e_2) = \{h_1, h_4\}, F(e_3) = \{h_1, h_2, h_3, h_6, h_7\}, F(e_4) = \{h_1, h_6, h_7\}\}$, $(G, B) = \{G(e_1) = \{h_1, h_2, h_4\}, G(e_2) = \{h_1, h_4\}, G(e_3) = \{h_1, h_2, h_7\}, G(e_4) = \{h_1, h_6\}, G(e_5) = \{h_1, h_4\}\}$. Clearly $A \subseteq B$ and $G(a) \subseteq F(a)$ for all $a \in A$. Therefore $(F, A)\widehat{\subset}(G, B)$.

Definition 5. [3] Let U be an initial universe set, E be the set of parameters, and $A \subseteq E$.

(a) (F, A) is called a relative null soft set (with respect to the parameter set A), denoted by $\emptyset_A$, if $F(a) = \emptyset$ for all $a \in A$.
(b) (G, A) is called a relative whole soft set (with respect to the parameter set A), denoted by $\mathfrak{U}_A$, if $G(e) = U$ for all $e \in A$.

The relative whole soft set with respect to the set of parameters E is called the *absolute soft set* over U and simply denoted by $\mathfrak{U}_E$. In a similar way, the relative null soft set with respect to E is called the *null soft set* over U and is denoted by $\emptyset_E$.

We shall denote by $\emptyset_\emptyset$ the unique soft set over U with an empty parameter set, which is called the *empty soft set* over U. Note that $\emptyset_\emptyset$ and $\emptyset_A$ are different soft sets over U and $\emptyset_\emptyset \widetilde{\subset} \emptyset_A \widetilde{\subset} (F, A) \widetilde{\subset} \mathfrak{U}_A \widetilde{\subset} \mathfrak{U}_E$ for all soft set (F, A) over U.

Definition 6. [3] Let (F, A) and (G, B) be two soft sets over the same universe U, such that $A \cap B \neq \emptyset$. The *restricted union* of (F, A) and (G, B) is denoted by $(F, A) \cup_{\mathcal{R}} (G, B)$ and is defined as $(F, A) \cup_{\mathcal{R}} (G, B) = (H, C)$, where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cup G(e)$.

If $A \cap B = \emptyset$, then $(F, A) \cup_{\mathcal{R}} (G, B) = \emptyset_\emptyset$.

Definition 7. [3] Let (F, A) and (G, B) be two soft sets over the same universe U such that $A \cap B \neq \emptyset$. The *restricted intersection* of (F, A) and (G, B) is denoted by $(F, A) \cap_{\mathcal{R}} (G, B)$ and is defined as $(F, A) \cap_{\mathcal{R}} (G, B) = (H, A \cap B)$ where $H(e) = F(e) \cap G(e)$ for all $e \in A \cap B$.

If $A \cap B = \emptyset$ then $(F, A) \cap_{\mathcal{R}} (G, B) = \emptyset_\emptyset$.

Definition 8. [3] Extended *union of two soft sets* (F, A) and (G, B) over the common universe U is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cup G(e) & \text{if } e \in A \cap B \end{cases}$$

We write $(F, A) \cup_{\mathcal{E}} (G, B) = (H, C)$.

Definition 9. [3] *The extended intersection* of two soft sets (F, A) and (G, B) over a common universe U, is the soft set (H, C) where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cap G(e) & \text{if } e \in A \cap B \end{cases}$$

We write $(F, A) \cap_{\mathcal{E}} (G, B) = (H, C)$.

Definition 10. [16] Let (F, A) and (G, B) be any two soft sets over a common universe U. Then the basic union of (F, A) and (G, B) is defined as the soft set $(H, C) = (F, A) \vee (G, B)$, where $C = A \times B$, and $H(a, b) = F(a) \cup G(b)$ for all $(a, b) \in A \times B$.

Definition 11. [16] Let (F, A) and (G, B) be any two soft sets over a common universe U. Then the basic intersection of (F, A) and (G, B) is defined as the soft set $(H, C) = (F, A) \wedge (G, B)$, where $C = A \times B$, and $H(a, b) = F(a) \cap G(b)$ for all $(a, b) \in A \times B$.

Definition 12. [14] Let E be a set of parameters and $A, B \subseteq E$. For $(a, b) \in A \times B$, (a and b) is called the conjunction parameter of (a, b), and (a or b) is called the disjunction parameter of ordered pair (a, b). These denoted by $(a \wedge b)$ and $(a \vee b)$ respectively.

We denote

$$A \otimes B = \{(a \wedge b) : (a, b) \in A \times B\}$$