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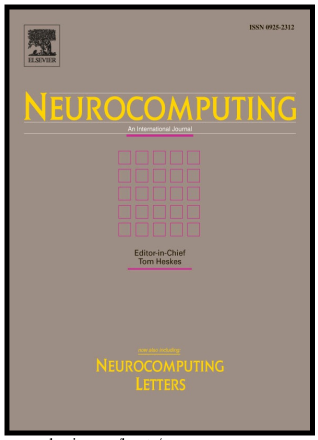

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Multistability of Recurrent Neural Networks with Time-varying Delays and the Nonincreasing Activation Function

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Abstract

In this paper, we are concerned with a class of recurrent neural networks (RNNs) with nonincreasing activation function. First, based on the fixed point theorem, it is shown that under some conditions, such an n-dimensional neural network with nondecreasing activation function can have at least $(4k + 3)^n$ equilibrium points. Then, it proves that there is only $(4k + 3)^n$ equilibria under some conditions, among which $(2k + 2)^n$ equilibria are locally stable. Besides, by analysis and study of RNNs with nondecreasing activation function, we can also obtain the same number of equilibria for RNNs with nonincreasing activation function. Finally, two simulation examples are given to show effectiveness of the obtained results.

Keywords: Multistability; Nonincreasing activation function; Nondecreasing activation function; Recurrent neural networks

1. Introduction

Dynamic behavior of equilibrium point has been one of the important contents of the Lyapunov theory, and has also been multistability concerned topic. Multistability study are beginning from multiple attractors analysis, which is closely related with the dynamic behavior of equilibrium point. In recent years, with the development of multistability and neural network, many efforts have been made on the applications such as signal processing, associative memories,image processing, pattern recognition, optimization problems, and so on (see [1-6]). Such applications also rely heavily on the dynamical properties of neural network systems. With the development of the application, it remains to be further indepth study dynamics behavior of neural network.

For the existence of equilibrium in selecting subset, the existing literature often adopts two kinds of methods. The two methods are respectively based on Banach fixed point and Brouwer fixed point. Adopting the Banach fixed point method, there is a strong constraints for the subarea. If the selected area is too large, it may not satisfy contraction; however, if the selected area is too small, it may not satisfy the condition of self-mapping. Using the other fixed point method, the exact number of the fixed point is not clear. Therefore, in order to make up for these deficiencies, it is necessary to combine the two methods.

Based on the Banach fixed point, it was shown that cellular neural networks could have 3^n memory patterns, of which 2^n were locally exponentially stably in [7]. In order to increase storage capacity, a class of discontinuous activation functions were introduced in [8], the coexistence of $(4k - 1)^n$ locally stably equilibrium points were derived. Multistability of recurrent neural networks (RNNs) with activation function symmetrical about the origin on the phase plane were also investigated in [9], in which new criteria on the multistability of neural networks were proposed. Also in [10-14], some multistability properties of neural networks were investigated, and some sufficient conditions were proposed to ensure system multistability.

Based on the Brouwer fixed point, in [15], the coexistence of multiple equilibrium points was investigated based on the geometrical configuration of the Fermi activation functions. In order to expand application scope, a class of nonsmooth activation functions were introduced in high-order neural networks in [16], and it was shown that the coexistence of 3^n equilibrium points and the local stability of 2^n equilibrium points. In addition, in [17], neural networks with discontinuous nonmonotonic piecewise linear activation functions could have at least 5^n equilibrium points, 3^n of which were locally stable and the others were unstable. For more references, please refer to [18-20] and so on.

In addition, the neural networks with concave-convex characteristics were investigated in [21-22], and it was shown that method based on a kind of other types of fixed point is given in these paper. In [23], multistable property of neural networks were addressed and this provided a method based on dynamic analysis, which had been applied to study the convergence of equilibria. Of course, dynamics analysis method combined with the fixed point method, also could deal with the coexistence of multistability, for example in [24-29] and so on. To explore multistability, these methods from [30-31] were also very valuable.

As far as we know, the type of activation functions plays a

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