Neurocomputing ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Sampling control on collaborative flocking motion of discrete-time system with time-delays

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ARTICLE INFO

Article history:
Received 2 July 2015
Received in revised form
2 March 2016
Accepted 25 July 2016
Communicated by Tao Li

Keywords:
Discrete-time system
Collaborative flocking
Multi-rate
Sampling control
Time-delay

ABSTRACT

Sampling control of multi-rate for flocking motion is researched in this paper. For discrete-time multi-agent systems with communication delays, a distributed sampling control protocol applying individual local information is proposed. Basing on Lyapunov stability theory, a energy function is designed, and collaborative flocking motion of multi-agent systems is studied. Finally, simulation examples show the effectiveness of the proposed multi-rate sampling protocol.

1. Introduction

Flocking, as a core problem in the complexity science, is essentially a kind of bionics method. Without centralized control, flocking motion produces macroscopic synchronous effect by mutual perception, which enhances their abilities of searching for food and avoiding predators. In the nature, flocking behavior exists widely, which is collaborative group behavior in a certain gathering way. In the military, flocking can combat replacing the army in order to reduce casualties.

With the development of biology, computer, control science, and artificial intelligence, flocking problem has drawn increasing attention of many scholars recently. In 1987, a computer model, imitating biological aggregation behavior, which contain gather, separation and adjustment three rules, is proposed by Reynolds [1]. Tanner et al. [2,3] firstly provide a theoretical explanation for the computer model, for the fixed and dynamic topology, an effective control protocol is proposed by Tanner. The local distributed protocol, usable the multi-agent systems to achieve flocking motion without collisions, is designed by Olfati-Saber [4,5]. As a kind of flocking motion, consensus of multi-agent systems has been studied deeply. Consensus of a heterogeneous multi-agent system with input saturation is studied in [6]. Reference [7] researches bounded consensus algorithms for multi-agent systems in directed networks. Finite-time consensus algorithm for multi-agent systems with double-integrator dynamics is presented in [8], distributed adaptive control for synchronization in directed complex networks is studied in [9]. Consensus of high-order multi-agent systems is analyzed in switching topologies [10], the observer-based consensus for nonlinear multi-agent systems is discussed with intermittent communication [11]. Dynamical consensus seeking of second-order multi-agent systems with communication delays is studied [12,13]. Flocking control algorithms are taken into account for multi-agent systems with communication time-delays and switching topology [14–16].

Up to now, most researches focus on the continuous system. The study of discrete system is more and more important with the rapid development of digital communication technology. In this aspect, Vicsek et al. [17] have proposed a discrete system model, where the velocity of agents can asymptotically converge to consistent state. For the communication delay problem, basing on frequency analysis and time analysis method, consistency of discrete-time systems with time delays is researched [18–20].

Research on multi-rate sampling begins 1950s. Using Kranc operator and sampler decomposition method, Kranc [21] solved the different sample frequency problem in the input and output sample. But when the sampling rates are more than two kinds, the complexity of the mathematical model makes traditional transfer function restricted in the application. Araki and Yamamoto [22] presented a complete state space description of the multi-rate sampling system. Using two different lifting technology, Yanamoto [23] put forward to the function space model of the multi-rate

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http://dx.doi.org/10.1016/j.neucom.2016.07.041
0925-2312/

sampling system. However, up to now, the results of multi-rate sampling control of networked system with communication time-delays have not been reported.

The main objective of this paper is to research multi-rate sampling control of collaborative flocking motion with time delays. The rest of the paper is organized as follows. In second part, problem description and preliminaries are given. The main results are discussed in third part. In fourth part, simulation examples are used to illustrate the correct of theoretical results. Conclusions are finally drawn in the fifth part.

2. Problem description and Preliminaries

2.1. Algebraic graph theory

Assume that the system consist of n agents. Let $G=(V,E,A)$ be a graph consisting of $V=\{v_1,v_2,\ldots,v_n\}$ with n agents and $E\subseteq V\times V$. $A=[a_{ij}]$ is a adjacency matrix of the graph G. The relationships among agents are described by the graph without self-loops. Moreover, the adjacency element $a_{ij}>0$ when $(v_i,v_j)\in E$, otherwise, $a_{ij}=0$. The set of neighbors of v_i is denoted by $N_i=\{v_j\in V, a_{ij}>0\}$. The Laplacian matrix of the graph G is defined as $L=D-A$, where $D=\mathrm{diag}(deg_{in}(v_1),deg_{in}(v_2),\ldots deg_{in}(v_n))$. Suppose that n eigenvalues of the Laplacian matrix are $\lambda_1(L)\le\lambda_2(L)\le\cdots\lambda_n(L)$. According to definition, there are $\lambda_1(L)=0$ and $LI=\lambda_1(L)I$, where $I=[1,1\ldots,1]^T$. Moreover, if the graph is connected, we can obtain $\lambda_2(L)>0$. Corresponding m dimensional Laplacian matrix can be rewritten as $\hat{L}=L\otimes I_m$, where I_m means unit matrix and $\otimes$ means Kronecker product.

2.2. Problem formulation

Basing on network topology $G=(V,E,A)$, there are n agents moving in m dimensional space. Dynamic equations of the ith agent are

$$\dot{q}_i(t)=p_i(t),\quad \dot{p}_i(t)=u_i(t).$$

With the development of digital communication technology, the application of discrete-time system is getting more and more widely. The discrete system of this dynamics can be described as

$$\begin{cases} q_i[k+1]=q_i[k]+T_k p_i[k]+\dfrac{T_k^2}{2}u_i[k],\\ p_i[k+1]=p_i[k]+T_k u_i[k], \end{cases} \tag{1}$$

where $q_i[k]$, $p_i[k]$ and $u_i[k]$ are the position vector, velocity vector and control input of the ith agent respectively. k is a discrete time sequence and T_k is the sampling period. In addition, assume that there is a leader in multi-agent systems, and the dynamic equations of the leader are

$$\begin{cases} q_r[k+1]=q_r[k]+T_k p_r[k],\\ p_r[k+1]=p_r[k], \end{cases} \tag{2}$$

where $q_r[k]$ and $p_r[k]$ are the leaders position vector and velocity vector. Moreover, this paper mainly focuses on the leaders uniform movement.

Definition 1. Potential function U_{ij} about relative distance $\| q_j(t)-q_i(t)\|$ between i and j is a differentiable, non-negative, unbounded function and satisfies the following two properties:

(1) U_{ij} tends to infinity when the distance $\| q_j(t)-q_i(t)\|$ tends to zero.
(2) U_{ij} can obtain the unique minimum at a desirable distance.

The potential function is applied in the paper [9], for example

$$U_{ij}(\| q_i-q_j\|)=c\left(\frac{d^2}{\| q_j-q_i\|^2}+\ln(\| q_j-q_i\|^2)\right), \tag{3}$$

where $c,d>0$. Let $\psi(z)=c\left(\frac{d^2}{z^2}+\ln z^2\right)$, thus $\phi(z)=\frac{d\psi(z)}{dz}=\frac{2c(z^2-d^2)}{z^3}$. The formula shows that U_{ij} obtains the unique minimum when $\| q_j(t)-q_i(t)\|=d$. d is named balanced distance. They will attract each other when the distance between agents is larger than d, on the contrary, mutually excluding.

Definition 2. A group of agents can asymptotically achieve flocking motion if their velocities asymptotically converge to consistency and the distances are remained constant between any two agents without collision.

2.3. Flocking motion algorithm

The aim is to design an effective distributed control protocol, which makes the system achieve flocking motion. In order to solve the problem, the following control protocol is designed:

$$u_i(k)=\begin{cases} \dfrac{1}{T_k}(T_k-\tau)\xi_i[k-1]+\dfrac{1}{T_k}\tau\xi_i[k], & t\in\left[kT_k,kT_k+\tau\right),\\ \dfrac{1}{T_k}(T_k-\tau)\xi_i[k]+\dfrac{1}{T_k}\tau\xi_i[k-1], & t\in\left[kT_k+\tau,kT_k+T_k\right), \end{cases} \tag{4}$$

$$\xi_i[k]=\sum_{j\in N_i}a_{ij}\phi(\| q_j[k]-q_i[k]\|)\vec{n}_{ij}+k\sum_{j\in N_i}a_{ij}(p_j[k]-p_i[k]) + h_i c_2(p_r[k]-p_i[k]), \tag{5}$$

where $k>0$ and $c_2>0$. $h_i=1$ means the ith agent can obtain the information from the leader; $h_i=0$ means the agent can not get the information. τ is communication time-delay between the ith agent and the jth agent and satisfies $0<\tau<T_k$. If the group meets $\lim_{k\to\infty}\| q_i[k]-q_j[k]\|=d$ and $\lim_{k\to\infty}\| p_i[k]-p_j[k]\|\to 0$ with any initial state, we can call the distributed control protocol is correct and effective.

3. Main results

Theorem 1. *Consider discrete-time multi-agent systems with communication time-delays. The dynamic behaviors of the agents are described by Eq. (1) and (2). Then, control protocol (4) and (5) can solve the collaborative flocking motion problem. Moreover, all agents can asymptotically achieve consistent velocities, and the distances will remain stable between any two agents without collision.*

Proof. Define the following variable:

$$\begin{cases} \tilde{q}_i[k]=q_i[k]-q_r[k],\\ \tilde{p}_i[k]=p_i[k]-p_r[k]. \end{cases} \tag{6}$$

By substituting Eqs. (1) and (2) into Eq. (6), we can obtain

$$\begin{cases} \tilde{q}_i[k+1]=\tilde{q}_i[k]+T_k\tilde{p}_i[k]+\dfrac{T_k^2}{2}u_i[k],\\ \tilde{p}_i[k+1]=\tilde{p}_i[k]+T_k u_i[k]. \end{cases} \tag{7}$$

(I) When the communication delay is ignored, because $\tau=0$, $u_i[k]=\xi_i[k]$. The protocol is

Please cite this article as: H. Fujun, et al., Sampling control on collaborative flocking motion of discrete-time system with time-delays, Neurocomputing (2016), http://dx.doi.org/10.1016/j.neucom.2016.07.041