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Learning coherent vector fields for robust point matching under manifold regularization

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ABSTRACT

In this paper, we propose a robust method for coherent vector field learning with outliers (mismatches) using manifold regularization, called manifold regularized coherent vector field (MRCVF). The method could remove outliers from inliers (correct matches) and learn coherent vector fields fitting for the inliers with graph Laplacian constraint. In the proposed method, we first formulate the point matching problem as learning a corresponding vector field based on a mixture model (MM). Manifold regularization term is added to preserve the intrinsic geometry of the mapped point set of vector fields. More specially, the optimal mapping function is obtained by solving a weighted Laplacian regularized least squares (LapRLS) in a reproducing kernel Hilbert space (RKHS) with a matrix-valued kernel. Moreover, we use the Expectation Maximization (EM) optimization algorithm to update the unknown parameters in each iteration. The experimental results on the synthetic data set, real image data sets, and non-rigid images quantitatively demonstrate that our proposed method is robust to outliers, and it outperforms several state-of-the-art methods in most scenarios.

1. Introduction

Point matching problem is a fundamental problem and plays a significant role in computer vision, signal processing, and pattern recognition [1–6], and it frequently arises in many applications, such as image registration, medical imaging, 3D reconstruction, image stitching, and object recognition.

The goal of the matching task is to distinguish inliers from outliers between given two point sets where each point set is captured from an image by a certain local feature extractor (e.g., SIFT [7], SURF [8,9]). However, the matching problem has several challenges: (1) initial correspondence set is usually contaminated by outliers (false matches or mismatches) after matching feature point pairs using similarity based method such as Best Bin First (BBF) [7], (2) the matching problem is an ill-posed problem and needs a constraint to preserve the intrinsic geometry of point set, (3) the transformation between point sets can be linear (e.g., translation, similarity, affine) or non-linear (e.g., quadratic, non-rigid), note that the latter one is hard to solve.

Many algorithms exist for point matching and try to address the above challenges. The most popular algorithm in the field is RANdom SAmple Consensus (RANSAC) [10], it repeatedly generates a hypothetical model from a small correspondence set, and then verifies each model on the whole set to select the best one. However, limitation occurs when facing non-linear transformations. To overcome those limitations, many progressive RANSAC algorithms have been developed, such as maximum likelihood estimation sample consensus (MLESAC) [11], progressive sample consensus (PROSAC) [12], non-rigid RANSAC [13]. It is worth noting that Sunglok et al. [14] has evaluated the performance of RANSAC algorithm family.

From the iterative point matching based methods [15,16], correct matches can be identified. Further, from the perspective of motion coherence (i.e., spatial coherence), the floating point set is moved to the target point set as close as possible by a set of smooth mapping functions. Some state-of-the-art methods are based on this motion field coherence theory (MCT) [17], such as coherent point drift (CPD) [18], Gaussian mixture model and thin-plate spline (GMM–TPS) [19], vector field consensus (VFC) [20,3,21], mixture of asymmetric Gaussian (MoAG) model [22,23], robust L_2E estimation [24], and context-aware Gaussian fields criterion (CA-LapGF) [25]. More specifically, the non-rigid transformation is parameterized by radial basis function (RBF), such as thin-plate spline (TPS), and Gaussian RBF (GRBF). Finally, outliers would be rejected as well as possible after learning a coherent motion field from point set pairs.

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Moreover, a topological clustering algorithm [26] was proposed and used to filter out mismatches. With this method, outliers can be identified and rejected by checking the consistency of topological relationships between matched regions in the image pair. The support vector machine regression method was used to identify the point correspondences and remove outliers (ICF) [27].

In this paper, we focus on identifying and removing outliers from point set matching as well as possible based on vector field learning (VFL). More specially, we first formulate the point matching as learning a coherent vector field mapping function, and then use the manifold regularization to constrain the vector field with preserving the intrinsic geometry. Our contribution in this paper includes the following two aspects. (1) We introduce the well-known manifold regularization framework for learning coherent vector fields with outliers. (2) Based on the MCT point matching model, we propose manifold regularized coherent vector field learning method (namely manifold regularized coherent vector field, MRCVF) for robust point matching, which can improve the matching accuracy compared to state-of-the-art methods. It is worth noting that our MRCVF is based on VFL method such as VFC [20], and the motivation derives from (1) the initial correspondence set contaminated by outliers, and (2) the natural property of manifold regularization.

The remainder of the paper is organized as follows. In Section 2 we first present the coherent vector field learning algorithm more formally and profoundly using manifold regularization constraint. In Section 3 we evaluate the proposed algorithm by some experiments on the public data set. In Section 4 we give a brief discussion and conclusion.

2. Methods

2.1. Vector field learning

Let us recall the familiar vector field learning briefly. Let input point set be $\mathbf{X}$ and output point set be $\mathbf{Y}$, then given a finite training set of labeled correspondences with some unknown outliers $S=\{(x_i, y_i)\}_{i=1}^N$. We define a mapping function f from a structured input space $\mathcal{X}\in\mathbb{R}^A$ to a structured output space $\mathcal{Y}\in\mathbb{R}^B$ from labeled examples S, then our task is to learn $f:\mathcal{X}\mapsto\mathcal{Y}$, i.e., $y_i=f(x_i)$ and identify the inliers (namely remove outliers), where $f\in\mathcal{H}$, and $\mathcal{H}$ is a reproducing kernel Hilbert space. Let $k: \mathcal{X}\times\mathcal{X}\mapsto\mathbb{R}$ be a standard Mercer kernel with an associated RKHS family of functions $\mathcal{H}_K: \mathcal{X}\mapsto\mathbb{R}$ with the corresponding norm $\|\cdot\|_{\mathcal{H}}$. Then the optimal mapping function f can be solved by minimizing the following Tikhonov regularized [28] optimization problem,

$$f^\star = \arg\min_{f\in\mathcal{H}_k}\frac{1}{N}\sum_{i=1}^N \| y_i - f(x_i)\|^2 + \lambda_1\|f\|_{\mathcal{H}}^2 \tag{1}$$

where the solution of f can be expressed by the classical Representer theorem [29] with finite dimensional coefficients $\boldsymbol{\alpha}=[\alpha_1,\dots,\alpha_N]^T$, i.e., $f^\star(x)=\sum_{i=1}^N\alpha_i k(x,x_i)$ with a linear system $(\boldsymbol{K}+\lambda NI)\boldsymbol{\alpha}=\mathbf{Y}$, where $\boldsymbol{K}$ is a positive semi-definite Gram matrix with $\boldsymbol{K}(i,j)=k(x_i,x_j)$, $\lambda_1>0$ is a trade-off parameter, I denotes the identity matrix.

2.2. Manifold regularized coherent vector field

In Manifold Regularization framework [30,31], an additional penalty term $\|f\|_{\mathcal{I}}^2$ is used to penalize f along a low dimensional manifold. Thus we can learn coherent vector fields under manifold regularization by minimizing the following extension of Eq. (1),

$$f^\star = \arg\min_{f\in\mathcal{H}_k}\frac{1}{N}\sum_{i=1}^N \| y_i - f(x_i)\|^2 + \lambda_1\| f\|_{\mathcal{H}}^2 + \lambda_2\|f\|_{\mathcal{I}}^2 \tag{2}$$

where λ_1 controls the complexity of the mapping function in the ambient space while λ_2 controls the complexity of the mapping function in the intrinsic geometry.

More specially, Let $\boldsymbol{W}$ be a nearest-neighbor graph which serves as a discrete probe for the geometric structure of the data, then the graph Laplacian $\boldsymbol{L}=\boldsymbol{D}-\boldsymbol{W}$ provides a natural intrinsic measure for simplicity of data-dependent smoothness,

$$\|f\|_{\mathcal{I}}^2 = \mathbf{f}^T\boldsymbol{L}\mathbf{f} = \frac{1}{2}\sum_{i,j=1}^N W_{ij}(f(x_i)-f(x_j))^2 \tag{3}$$

where $\mathbf{f}=[f(x_1),\dots,f(x_N)]$, note that $\boldsymbol{D}$ is a diagonal matrix with elements $\boldsymbol{D}_{ii}=\sum_{j=1}^N W_{ij}$. The solution of coherent vector fields will be discussed later.

2.3. Learning coherent vector fields

Motivated by the sample consensus, the inliers can be fitted by a coherent vector field mapping. Thus we assume that the error between $\mathbf{Y}$ and $f(\mathbf{X})$ satisfies the following distributions,

$$\mathbf{Y}-f(\mathbf{X}) = \begin{cases}\epsilon_i\sim\mathcal{N}(0,\sigma^2 I) & \text{if inliers,}\\ \epsilon_o\sim\dfrac{1}{u} & \text{if outliers.}\end{cases} \tag{4}$$

where the error for inliers satisfies Gaussian distribution with zero mean and uniform standard deviation σ, while the error for outliers satisfies a uniform distribution $\frac{1}{u}$ with a positive constant u. Thus the error between observed input-output pairs is modeled as a mixture model of the Gaussian and uniform distributions [11,18,20,32],

$$p(S_i|\theta) = \gamma\frac{1}{(2\pi\sigma^2)^{\frac{D}{2}}}\exp\left(-\frac{\| y_i - f(x_i)\|^2}{2\sigma^2}\right) + (1-\gamma)\frac{1}{u} \tag{5}$$

where $0\le\gamma\le1$ is a mixing coefficient denoting the percentage of inliers, $\theta=\{f,\gamma,\sigma^2\}$ is the set of unknown parameters, and D denotes the dimension of data.

Moreover, to reduce over-fitting and preserve smoothness constraint, the prior of the coherent mapping function f under manifold regularization can be expressed as follows,

$$p(f)\propto\exp\left(-\lambda_1\|f\|_{\mathcal{H}}^2 - \lambda_2\|f\|_I^2\right) \tag{6}$$

According to Bayes' theorem, the posterior distribution $p(\theta|S)$ could be estimated by the given (5) and prior (6),

$$p(\theta|S)\propto\mathcal{L}(\theta|S)p(f) \tag{7}$$

where the likelihood $\mathcal{L}(\theta|S)=\prod_{i=1}^N p(S_i|\theta)$, and the optimal solution of θ is to estimate a maximum a posteriori (MAP).

Considering the complete-data with a latent variable z_i, where $z_i=0$ for outliers, and $z_i=1$ for inliers, then the objective function is an upper bound of the negative log-likelihood function of (7),

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