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Adaptive neural control for a class of stochastic non-strict-feedback nonlinear systems with time-delay ☆

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This paper addresses adaptive neural control for a class of non-strict-feedback stochastic nonlinear systems with time delays. An important structural property of radial basis function (RBF) neural networks (NNs) is introduced to overcome the design difficulty from the non-strict-feedback structure. The Lyapunov–Krasovskii functional is used for control design and stability analysis. Further, a backstepping-based adaptive neural control strategy is proposed. The suggested adaptive neural controller guarantees that all the closed-loop signals are semi-globally uniformly ultimately bounded (SGUUB) and the tracking error converges to a small neighborhood of the origin. Simulation results demonstrate the effectiveness of the proposed approach.

1. Introduction

During the past decades, many scholars have devoted much effort to approximation-based adaptive fuzzy or neural control for nonlinear systems. By their inherent ability in function approximation, neural networks (NNs) or fuzzy logical systems (FLSs) are used to model the unknown nonlinear functions in order to achieve control design for nonlinear systems. The work in [1] proposed an adaptive fuzzy control method for a class of nonlinear systems with unknown system functions. Some similar adaptive fuzzy control strategies were further proposed in [2–10] for nonlinear uncertain systems. On the other hand, a series of adaptive control design approaches were also developed for nonlinear systems by using RBF NNs as function approximators, for instance see [11–18] and the reference therein. Furthermore, adaptive neural or fuzzy control was discussed for multi-input and multi-output (MIMO) nonlinear strict-feedback systems in [18–20], respectively. In these researches, adaptive neural or fuzzy controllers are constructed recursively in the framework of the backstepping. Notice that time delay phenomena often occurs in many practical systems, such as physical, biological, and economical systems. The existence of time delays is usually a main reason of instability and deteriorative performance of the controlled systems. Therefore, many researchers further extended the approximation-based adaptive control from delay-free systems to delayed systems. Some significant results have been reported for deterministic systems, for example, [21–28]. Ge and Tee [27] presented approximation-based adaptive neural control for a class of MIMO nonlinear time-delay systems, which is in block-triangular form. Liu et al. [4] further considered the coupling effect of time delays and dead-zone in the control design procedure. At the same time, some results were obtained for stochastic time-delay systems, for example see [28–33]. Liu and Xie [33] presented adaptive neural control schemes for a class of stochastic high-order nonlinear systems with time-varying delays. However, all the aforementioned control strategies only focus on the nonlinear systems in strict-feedback form [34–36] or in pure-feedback form [37]. To relax the restriction on system structure, Chen et al. [38–40] developed a variable separation technique by using the monotonically increasing functions as the bounding functions. An adaptive fuzzy control scheme was presented for non-strict-feedback nonlinear systems, in which each subsystem function, i.e. $f_i(\cdot)$ contains all the state variables. Then, the control laws were proposed under the following assumptions: (i) the function $f_i(\cdot)$ must be bounded by a strictly increasing smooth function and (ii) the function $g_i(\cdot)$ does not contain the state variable x_k for $k \geq i+1$. In practice, it is difficult to check if these assumptions are satisfactory, particularly, when the system functions are unknown. In

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addition, the authors in [38,39] only studied the deterministic systems, but not the stochastic systems. The stochastic distribution was put into account in [40], while the diffusion terms $\psi(\cdot)$ s in those papers were supposed to be the function of the previous variable $\bar{x}_i$. When the diffusion terms $\psi(\cdot)$ in the ith subsystems is a nonlinear function of the whole state variables, how to design the virtual control signal α_i which is independent of the state variables x_j, $j = i+1, \ldots, n$ is the main difficulty to be overcome. In addition, to the best of our knowledge, so far, there has been no results about non-strict-feedback with time-delay to be reported.

Motivated by the aforementioned discussion, in this research we will consider adaptive neural control for a class of unknown nonlinear delayed stochastic systems in non-strict-feedback structure. The main differences between our work and the ones in [38–40] are that unlike [38–40]: (1) the system function $f_i(\cdot)s$ and the diffusion terms $\psi(\cdot)$ s are not required to be bounded by a strictly increasing smooth function and (2) the functions $g_i(\cdot)$ s contain all the state variables. The system (6) is thus more general than that in [38–40]. Therefore, the proposed control strategy is easier to be implemented in practice than the existing results. In the process of constructing adaptive neural controller, a characteristic of RBF NNs shown in Lemma 4 is utilized to deal with the system functions which contain the whole state variable. A Lyapunov–Krasovskii functional is used to compensate for the effect of nonlinear time-delay terms. An adaptive neural tracking control scheme has been developed. The proposed adaptive neural controller guarantees that all the signals in the closed-loop are bounded and the tracking error converges to a small neighborhood of the origin.

2. Preliminary knowledge and system formulation

For the purpose of introducing some useful concepts and lemmas, consider the following stochastic system:

$$dx = f(t,x)dt + h(t,x)dw, \quad \forall x \in R^n \tag{1}$$

where x is the state variable, w is r-dimensional independent standard Brownian motion and f and h are vector-value or matrix-value function with appropriate dimensions.

Definition 1. [41] For any given $V(t,x) \in C^{1,2}$, associated with the stochastic differential equation (1) define the operator L as follows:

$$LV = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f + \frac{1}{2}\mathrm{Tr}\left\{h^T \frac{\partial^2 V}{\partial x^2} h\right\} \tag{2}$$

where $\mathrm{Tr}(A)$ is the trace of A.

Definition 2. [42] The trajectory $\{x(t), t \le 0\}$ of stochastic system (1) is called SGUUB in p-th moment, if for some compact set $\Omega \in R^n$ and any initial state $x_0 = x(t_0)$, there exist a constant $\varepsilon > 0$, and time constant $T = T(\varepsilon, x_0)$ such that $E(|x(t)|^p) < \varepsilon$, for all $t > t_0 + T$. Especially, when $p = 2$, it is usually called SGUUB in mean square.

Lemma 1. *[43] Suppose that there exists a $C^{1,2}$ function $V(t,x)$: $R^+ \times R^n \to R^+$, two constants $C_1 > 0$ and $C_2 > 0$, class k_∞-functions $\bar{\alpha}_1$ and $\bar{\alpha}_2$ such that*

$$\bar{\alpha}_1(|x|) \le V(t,x) \le \bar{\alpha}_2(|x|)$$
$$LV(t,x) \le -C_1 V(t,x) + C_2 \tag{3}$$

for all $x \in R^n$ and $t > t_0$. Then, there is an unique strong solution of system (1) for each $x_0 \in R^n$ and it satisfies

$$E[V(t,x)] \le V(x_0)e^{-C_1 t} + \frac{C_2}{C_1} \quad \forall t > t_0$$

In the developed control design procedure, RBF NNs will be used to approximate the unknown nonlinear functions. A useful property of RBF NNs proposed in[44] is first listed as follows. It has been proved that with sufficiently large node number l, the RBF NNs $W^{*T}S(Z)$ can approximate any continuous function $f(Z)$ over a compact set $\Omega_z \subset R^q$, for given arbitrary accuracy $\varepsilon > 0$, such that

$$f(Z) = W^{*T}S(Z) + \delta(Z), \quad \forall z \in \Omega_z \subset R^q, \tag{4}$$

where $|\delta(Z)|$ is the error approximation satisfying $|\delta(Z)| \le \varepsilon$, $W^* = [w_1, w_2, \ldots, w_l] \in R^l$ denotes the ideal constant weight vector, and defined as

$$W^* = \arg\min_{W \in R^l}\{\sup_{Z \in \Omega_Z}|f(Z) - W^T S(Z)|\}$$

and $S(Z) = [s_1(Z), \ldots, s_l(Z)]$ stands for the basis function vector, with $l > 0$ being the number of the neural networks nodes, and $s_i(Z)$ are chosen as Gaussian function, namely,

$$s_i(Z) = \exp\left[-\frac{(Z-\mu_i)^T(Z-\mu_i)}{\eta_i^2}\right], \quad i = 1, \ldots, l \tag{5}$$

where $\mu_i = [\mu_{i1}, \ldots, \mu_{iq}]^T$ is the center of the receptive field and η_i is the width of Gaussian function.

Lemma 2. *[45] For any real-valued continuous function $f(x,y)$, where $(x,y) \in R^m \times R^n$, there are smooth scalar functions $a(x) \ge 0$, $b(y) \ge 0$ such that*

$$|f(x,y)| \le a(x) + b(y).$$

Lemma 3. *[46] For $1 \le i \le n$, define the set Ω_{z_i} as $\Omega_{z_i} \triangleq \{z_i \,\|\, z_i| \ge 0.8814\nu_i\}$. Then, for $z_i \in \Omega_{z_i}$, the inequality $(1 - 4\tanh^4(z_i/\nu_i)) \le 0$ holds, where ν_i is any positive constant.*

Lemma 4. *Let $\bar{x}_q = [x_1, x_2, \ldots, x_q]^T$ and $S(\bar{x}_q) = [s_1(\bar{x}_q), \ldots, s_l(\bar{x}_q)]^T$ be the basis function vector. Then, for any positive integer $k \le q$, the following inequality holds:*

$$\| S(\bar{x}_q)\|^2 \le \| S(\bar{x}_k)\|^2.$$

The proof is omitted.

Remark 1. This lemma provides a simple but useful characteristic of RBF NN. By which adaptive neural backstepping design method can be extended to the non-strict-feedback system (6) easily.

In this work, we consider the following non-strict-feedback stochastic nonlinear systems with time delays:

$$dx_i = [g_i(x(t))x_{i+1} + f_i(x(t)) + q_i(x(t-\tau_i))]dt + \varphi_i^T(x(t))dw,$$
$$i = 1, \ldots, n-1,$$
$$dx_n = [g_u(x(t))u + f_n(x(t)) + q_n(x(t-\tau_n))]dt + \varphi_n^T(x(t))dw,$$
$$y = x_1(t), \tag{6}$$

where $x = [x_1, \ldots, x_n]^T \in R^n$ and $y \in R$ are system state variable and output, respectively. The mappings f_i: $R^n \to R$, g_i: $R^n \to R$, q_i: $R^n \to R$ and φ_i: $R^n \to R^r$ are assumed to be unknown smooth functions with $f_i(0) = 0$ and $q_i(0) = 0$, $\varphi_i(0) = 0$ and τ_i are unknown constant time delays. The control objective is to structure an adaptive NNs controller for system (6), such that (i) all the closed-loop signals remain uniformly ultimately bounded, and (ii) output y follows a desired reference signal y_d. To this purpose, some assumptions are introduced as follows.

Assumption 1. The sign of g_i does not change, and there exist constants b_m and b_M such that

$$0 < b_m \le |g_i(x)| \le b_M < \infty, \quad i = 1, \ldots, n, \tag{7}$$

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