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Event-based input-constrained nonlinear H∞ state feedback with adaptive critic and neural implementation ☆

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ABSTRACT

In this paper, the continuous-time input-constrained nonlinear H_∞ state feedback control under event-based environment is investigated with adaptive critic designs and neural network implementation. The nonlinear H_∞ control issue is regarded as a two-player zero-sum game that requires solving the Hamilton–Jacobi–Isaacs equation and the adaptive critic learning (ACL) method is adopted toward the event-based constrained optimal regulation. The novelty lies in that the event-based design framework is combined with the ACL technique, thereby carrying out the input-constrained nonlinear H_∞ state feedback via adopting a non-quadratic utility function. The event-based optimal control law and the time-based worst-case disturbance law are derived approximately, by training an artificial neural network called a critic and eventually learning the optimal weight vector. Under the action of the event-based state feedback controller, the closed-loop system is constructed with uniformly ultimately bounded stability analysis. Simulation studies are included to verify the theoretical results as well as to illustrate the event-based H_∞ control performance.

1. Introduction

When coping with nonlinear optimal regulation designs during various control practices, we always encounter the difficulty of solving the Hamilton–Jacobi–Bellman (HJB) equation [1]. In particular, the input-constrained control design is a more complicated problem than the traditional unconstrained design task [2,3]. Though dynamic programming is deemed as a basic strategy to handle optimal control problems, there still exists a serious issue called the "curse of dimensionality". Similarly, from the point of minimax optimization, the H_∞ control problem can be formulated as a two-player zero-sum differential game. In order to obtain a controller that minimizes the cost function in the worst-case disturbance, it is required to find the Nash equilibrium solution by dealing with the Hamilton–Jacobi–Isaacs (HJI) equation. Nevertheless, it is also intractable to gain an analytic solution of the HJI equation in case of nonlinear systems. Therefore, various approximate methods have been proposed to overcome the difficulty in handling the HJB and HJI equations. Among that, by involving neural networks for function approximation, the adaptive or approximate dynamic programming (ADP) was founded by Werbos [4] to solve optimal control problems forward-in-time. As Lewis and Liu [5] stated, there exists a fundamental idea in ADP which is similar as designing advanced adaptive systems with neural network technique (see, e.g., [6–9]). Note that therein, various nonlinearities, such as uncertain dynamics, input saturation, and dead-zone input, were considered and handled by constructing powerfully adaptive and neural systems. Hence, it is greatly important to understand and construct more intelligent adaptive systems, especially optimal adaptive systems, with the help of ADP methodology. Actually, it is observed that ADP and related fields have gained much development in various topics, such as adaptive optimal regulation [10–13], optimal tracking control [14–17], robust optimal control [18–20] and so on. Recently, the nonlinear H_∞ control and the non-zero-sum game have also been paid special attention under ADP framework [21–26]. For example, Abu-Khalaf

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et al. [21] performed policy iterations on both players (constrained control law and disturbance law), so as to solve the HJI equation of H_∞ state feedback control problem with input saturation. Liu et al. [25] studied the neural-network-based zero-sum game for a class of nonlinear discrete-time systems via the iterative ADP algorithm. Note that the above results are all derived with the traditional time-based design manner.

When designing automatic control systems, especially distributed and networked systems, the event-based approach has gained much attention in the last decade, since people can benefit greatly from it in terms of decreasing the computation complexity and enhancing the control efficiency [27–34]. Eqtami et al. [28] proposed an event-based strategy for state feedback control of discrete-time nonlinear systems. Tallapragada and Chopra [29,30] developed an event-based control algorithm for trajectory tracking of continuous-time nonlinear systems. Li et al. [31,32] and Dong et al. [33] studied the event-triggered state estimation and synchronization control for complex networks with the involvement of time-varying delays, uncertain inner coupling, and state-dependent noises. Ma et al. [34] constructed both centralized and decentralized event-triggered control protocols for group consensus to cope with energy consumption and communication constraint that may be encountered in physical implementations. In particular, the combination of event-based mechanism and ADP method provides a novel channel for achieving advanced nonlinear optimal control with adaptivity [35–38]. Among them, Vamvoudakis [35] originally proposed an event-based adaptive optimal control strategy for continuous-time affine nonlinear systems. Note that under the new framework, the ADP-based controller is updated only when an event is triggered, thereby reducing the computational burden of learning and control. However, the existing research is conducted either for nonlinear regulation problem, or without considering the input constraints, which calls for an extension to input-constrained zero-sum game design under event-based formulation. This motivates our research. Note that the main difficulty and challenge of introducing the event-based framework is how to conduct the critic learning and analyze the closed-loop stability in case that the event-based state vector is taken into consideration.

This paper focuses on the event-based input-constrained nonlinear H_∞ state feedback control with the idea of ADP. In order to emphasize the ability of adaptivity and self-learning, we call the ADP architecture established here as adaptive critic learning (ACL). The main contribution of this paper lies in that the event-based design framework is combined with the ACL technique, so as to accomplish the input-constrained nonlinear H_∞ state feedback. The rest of this paper is organized as follows. A brief description of the input-constrained nonlinear H_∞ control problem is provided in Section 2. The ACL methodology for the event-based input-constrained nonlinear H_∞ control is developed in Section 3 with closed-loop stability analysis. The simulation studies and the concluding remarks are presented in Sections 4 and 5, respectively.

For convenience, the following notations will be utilized throughout the paper. $\mathbb{R}$ represents the set of all real numbers. $\mathbb{R}^n$ is the Euclidean space of all n-dimensional real vectors. $\mathbb{R}^{n\times m}$ is the space of all $n\times m$ real matrices. $\|\cdot\|$ denotes the vector norm of a vector in $\mathbb{R}^n$ or the matrix norm of a matrix in $\mathbb{R}^{n\times m}$. I_n represents the $n\times n$ identity matrix. $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ calculate the maximal and minimal eigenvalues of a matrix, respectively. C^κ denotes the class of functions having continuous κ-th derivative. Let Ω be a compact subset of $\mathbb{R}^n$ and $\Psi(\Omega)$ be the set of admissible controls on Ω. $\mathbb{N}=\{0,1,2,\dots\}$ denotes the set of all non-negative integers. In addition, the superscript "T" is taken for representing the transpose operation and $\nabla(\cdot)\triangleq\partial(\cdot)/\partial x$ is employed to denote the gradient operator.

2. Problem description of the input-constrained nonlinear H_∞ control problem

Let us consider a class of continuous-time nonlinear systems with external perturbations described by

$$\dot{x}(t)=f(x(t))+g(x(t))u(t)+h(x(t))v(t); \tag{1a}$$

$$z(t)=Q(x(t)), \tag{1b}$$

where $x(t)\in\Omega\subset\mathbb{R}^n$ is the state vector, $u(t)\in\Omega_u\subset\mathbb{R}^m$ is the control vector, $v(t)\in\mathbb{R}^q$ is the perturbation vector with $v(t)\in L_2[0,\infty)$, $z(t)\in\mathbb{R}^p$ is the objective output, and $f(\cdot)$, $g(\cdot)$, and $h(\cdot)$ are differentiable in their arguments with $f(0)=0$. The constrained control set is defined as $\Omega_u=\{u\in\mathbb{R}^m: |u_i|<\bar{u}_i, i=1,2,\dots,m\}$. We let $x(0)=x_0$ be the initial state and $x=0$ be the equilibrium point of the controlled plant.

Assumption 1. The system function $f(x)$ is Lipschitz continuous on a set Ω in $\mathbb{R}^n$ containing the origin and the system (1a) is controllable.

With Assumption 1, for nonlinear H_∞ control, it is aimed at obtaining a state feedback control law $u=u(x)$ such that the closed-loop form of system (1a) is asymptotically stable, and simultaneously, has L_2-gain no larger than ϱ, that is

$$\int_0^\infty\left(\|Q(x)\|^2+Y(u)\right)\mathrm{d}\tau\le\varrho^2\int_0^\infty v^{\mathrm{T}}(\tau)Pv(\tau)\mathrm{d}\tau, \tag{2}$$

where $\|Q(x)\|^2=x^{\mathrm{T}}(t)Qx(t)$ and Q and P are symmetric positive definite matrices with appropriate dimensions. If the condition (2) is satisfied, the closed-loop system is said to have L_2-gain no larger than ϱ. For unconstrained control problem, we often select a quadratic utility regarding to u as $Y(u)=u^{\mathrm{T}}(t)Ru(t)$ with R being a symmetric positive definite matrix. However, for input-constrained control problem, inspired by [2,21], a non-quadratic utility is adopted by choosing

$$Y(u)=2\int_0^u\varphi^{-\mathrm{T}}(\varsigma)R\mathrm{d}\varsigma, \tag{3}$$

where $\varphi(\cdot)\in\mathbb{R}^m$ is a m-dimensional function, $\varphi^{-\mathrm{T}}$ denotes $(\varphi^{-1})^{\mathrm{T}}$, and $\varphi^{-1}(\varsigma)=(\varphi_1^{-1}(\varsigma_1),\varphi_2^{-1}(\varsigma_2),\dots,\varphi_m^{-1}(\varsigma_m))^{\mathrm{T}}$. Meanwhile, $\varphi_i(\cdot)$ is a strictly monotonic odd function satisfying $|\varphi_i(\cdot)|<1$ $(i=1,2,\dots,m)$ and belonging to C^κ $(\kappa\ge1)$ and $L_2(\Omega)$.

Remark 1. It is important to point out that this kind of non-quadratic utility is a nominal choice in light of the literature, such as [2,21]. Clearly, $Y(u)$ is positive definite since $\varphi_i(\cdot)$ is a monotonic odd function, for instance, $\varphi_i(\cdot)=\tanh(\cdot)$.

As is known, the H_∞ control problem can be formulated as a two-player zero-sum differential game, where the control input is a minimizing player while the disturbance is a maximizing one [21,23,24,38]. Note that the solution of H_∞ control problem is the saddle point of zero-sum game theory, denoted as (u^*,v^*), where u^* and v^* are the optimal control and the worst-case disturbance, respectively.

Define the infinite horizon cost function as

$$J(x(t),u,v)=\int_t^\infty U(x(\tau),u(\tau),v(\tau))\mathrm{d}\tau, \tag{4}$$

where

$$U(x,u,v)=x^{\mathrm{T}}Qx+Y(u)-\varrho^2v^{\mathrm{T}}Pv$$

represents the utility function. For the two-player zero-sum differential game, our goal is to find the feedback saddle point solution (u^*,v^*), such that the following Nash condition holds:

$$J^*(x_0)=\min_u\max_v J(x_0,u,v)=\max_v\min_u J(x_0,u,v).$$