Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Full frequency de-noising method based on wavelet decomposition and noise-type detection

Wenyong Dong [a], Hong Ding [b,*]

[a] Computer School, Wuhan University, Wuhan, Hubei, PR China

[b] School of Mathematics and Computer Science, Guangxi Science and Technology Normal University, Liuzhou, PR China

ARTICLE INFO

Article history:
Received 15 July 2015
Received in revised form
8 June 2016
Accepted 22 June 2016
Communicated by R. Capobianco Guido

Keywords:
Noise-type detection
De-correlation
Adaptive wavelet threshold
Full frequency de-noising
Noise reduction

MSC Classification:
00-01
99-00

ABSTRACT

Traditional wavelet threshold de-noising techniques assume that noise is spread over a high frequency band of signal. However, it may not be true for some noise, and those methods rarely concern the noise in low frequency bands. This motivates us to study new methods to reduce noise in the whole wavelet frequency band. Thus, a new framework named Full Frequency band De-noising based on Noise-type Detection (FFD-ND) is proposed. In this framework, a noise type is detected by analyzing autocorrelation coefficients for different noise, and then noise reduction is performed both in low and high frequency band by using different thresholds for different noise models. To analyze the necessity for de-noising in low frequency bands, we firstly study the relationships between power spectral densities (PSDs) and wavelet decomposition scales for some noisy models, and find that it is not true for most of the noises and that PSDs decline with the reduction of wavelet decomposition scales. As for wavelet threshold value, we also find that it relies not only on wavelet decomposition scales but also on noise models. To adaptively determine the threshold value, we then propose an adaptive approach, in which the threshold value is functionally dependent on noise model, wavelet decomposition layers and other factors. The proposed approach can always achieve better performance with lower computation cost and fewer decomposition scales than a high frequency de-noising method. We also experimentally verify that the performance of our method for noise-type detection is super than the methods based on neural network.

1. Introduction

Due to the influence of many random factors, all kinds of signals obtained from a physical environment may contain disturbing noise, which may decrease the performance of visual and computerized analysis method [1–4]. Therefore, many de-noising techniques are proposed to overcome this problem [3–6]. A de-noising process can be described as to remove the noise but not to distort the quality of processed signal. Among all the de-noising methods, wavelet soft and hard threshold de-noising is the state-of-the-art one, which has been widely applied to many one- or two-dimensional signals [7–14]. The threshold can be roughly set in four ways: fixed, Steins Unbiased Risk Estimate, Heursure and minimax. As a view of mathematics, wavelet transform is regarded as an approximation method, and used to expand and approximate as a function on the wavelet basis in a particular space. Wavelet de-noising method is to eliminate the small coefficient in high frequency domain which is assumed to be associated to the noise, and the original signal can be obtained by the reconstruction algorithm using the noise free coefficients.

Traditional wavelet threshold de-noising techniques assume that the signal energy is mainly concentrated on low frequency domain, while noise energy exists in high frequency domain. Hence, the de-noising methods based on wavelet threshold only tend to handle the high frequency noise instead of the low frequency noise. Those methods have the following disadvantages: (1) do not consider the influence of the noise in the low frequency domain; (2) do not consider the value of wavelet threshold depended on the noise type. Thus, Mallat also pointed out that we should not ignore the noise influence in the low frequency domain in his early research [9]. In order to overcome above issues, a framework of full frequency noise reduction is proposed in this paper. In the proposed framework, the noise type in a signal is firstly detected. Then an adaptive threshold function is derived to choose an appropriate threshold to de-noise by using a full frequency noise reduction method.

This paper is organized as follows. Wavelet threshold de-noising method and noise type detection methods are reviewed in

* Corresponding author.
E-mail addresses: dwy@whu.edu.cn (W. Dong), dhong20123@163.com (H. Ding).

http://dx.doi.org/10.1016/j.neucom.2016.06.072
0925-2312/© 2016 Published by Elsevier B.V.

Section 2. The necessity of full frequency noise reduction is analyzed in Section 3. In Section 4, the computing methods of adaptive threshold for different noise are discussed, and a full frequency de-noising algorithm is described. The experiments are described in Section 5. Finally, the conclusions are drawn in Section 6.

2. Backgrounds on relevant works

2.1. Wavelet threshold de-noising methods

The coefficients of wavelet transform of signal are usually sparse, especially for noiseless signal because most of them are near zero. Therefore, coefficients with small magnitude can be considered as being from noise and should be set to zero. A wavelet de-nosing procedure is composed of the following process: (1) apply wavelet transform to the original signal to obtain wavelet coefficient series in different level; (2) select different thresholds for each level and remove the noise; and (3) inverse wavelet transform to obtain a de-noised signal.

There are four kinds of methods to determine the threshold: fixed threshold [10], rigorous sure threshold [11], Heursure threshold [15] and min–max threshold [16].

- Fixed threshold:
  This is the simplest wavelet threshold method by performing a wavelet transformation of the given data and then using the same threshold to deal with all the coefficients in the expansion. It uses the fixed threshold form given as:

$$\lambda = \sigma\sqrt{2\log N} \tag{1}$$

  where λ is the estimated threshold, N denotes the length of the analyzed signal and σ is given by the following equation:

$$\sigma = \frac{median(x)}{0.6745} \tag{2}$$

  where x is a median selection made on the detail coefficient of the analyzed signal.
- Rigorous sure threshold:
  Rigorous sure threshold, which describes a scheme that uses different thresholds for each resolution level, is proposed by Donoho. The threshold is given as:

$$\lambda_S = \arg\min_{0<\lambda<\lambda_u} sure\left(\lambda, \frac{S(a,b)}{\sigma}\right) \tag{3}$$

  where *sure* is defined as

$$sure(\lambda, X) = n - 2\cdot\Theta\left\{i: |X_i \le \lambda|\right\} + \left[\min(|X_i|, \lambda\right]^2 \tag{4}$$

  where the operator $\Theta(\cdot)$ returns the cardinality of the set $\left\{i: |X_i \le \lambda|\right\}$
- Heursure threshold:
  Heursure threshold is a combination of the first and second thresholds. If the signal-to-noise ratio is low, then fixed threshold is used. Otherwise, the latter is used.
- Min–max threshold:
  The min–max threshold value M proposed by Donoho consists of an optimal threshold derived from minimizing the constant term in an upper bound of the risk involved in the estimation. The proposed threshold depends on the available data and the noise level contaminating the signal.

2.2. Noise type detection methods

In prior research, most of the detection methods are target at image noise [17,18]. In [19], white noise verification is proposed based on wavelet de-correlation. After wavelet transform, noisy signal is divided into high and low frequency parts, the low frequency part mainly represents the trend or relevant part while the high frequency part represents the detail. If the wavelet coefficient is greater than the threshold, the coefficients are set to zeros, they should remain unchanged, and then the wavelet coefficients are reconstructed to obtain the estimated signal. This is called wavelet de-correlation. Based on the wavelet de-correlation, the noise type can be detected. Namely, by comparing the sum of the autocorrelation coefficient of the noisy signal with the sum of the autocorrelation coefficient of all noise, we can obtain that the noise corresponding to the largest coefficient is the one involved in the original noisy signal.

3. Why need full frequency de-noised?

According to the No Free Lunch theory, there is no a common de-noising threshold irrelevant to noise-type that can well handle all the noises. That is why we need to design different thresholds for various noises, which is supported by analyzing the characters of seven common noises, i.e., white, rand, color, pink, red, blue and violet noises.

3.1. Power spectrum distributions

The Welch method is used to estimate a noise power spectrum. Firstly the data is divided into separate or overlapping segments, and then each segment is multiplied by a window function and afterwards Discrete Fourier Transform is performed. From the power spectrum of the seven kinds of noise, we can find the following characters: (1) the PSDs of white and random noises remain similar in whole frequency bands; (2) the PSD of color noise is also similar in low and medium frequency bands, but they decreased when the frequency increases; (3) the PSD of pink noise is mainly distributed in low-mid frequency band, and with the increase of frequency, the power density of per octave decreases 3 db, which can be described by a logarithmic function; (4) the PSD of red noise is similar to the one of pink noise; and (5) the PSD of violet noise increases 6 db for per octave with the increase of frequency. They imply that we cannot use the same threshold value to de-noise for each noise in different frequency bands.

3.2. Energy distribution viewpoint

Seven kinds of noise are decomposed in multi-scale by using coif1 wavelet, and energy change at different scales is calculated. The energy formula is

$$E(x) = \sum_{t=1}^{m} (f_x(t))^2 \tag{5}$$

where E defines the energy, x is the wavelet decomposing scale, $f_x(t)$ is the subseries of x scale.

From Fig. 1(a) we can see that the energy of white, color, random, blue and purple noises, decreases with the increase of the scale; while red and pink noises have less energy in the high frequency bands and their energy changes slowly as the scale increases. Therefore, it can achieve good effect to de-noise the five kinds of noise in high frequency band. The energy of pink and red noises in the high frequency band is always small, thus it will not get an obvious effect to de-noise in a high frequency domain.