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# A simple and efficient real-coded genetic algorithm for constrained optimization

Yao-Chen Chuang[a], Chyi-Tsong Chen[a,b,*], Chyi Hwang[b]

[a] Department of Chemical Engineering, Feng Chia University, No. 100, Wenhwa Rd., Seatwen, Taichung 407, Taiwan
[b] Center of General Education, National Quemoy University, No. 1, University Rd., Jinning Township, Kinmen 892, Taiwan

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ABSTRACT

This paper presents a simple and efficient real-coded genetic algorithm (RCGA) for constrained real-parameter optimization. Different from some conventional RCGAs that operate evolutionary operators in a series framework, the proposed RCGA implements three specially designed evolutionary operators, named the ranking selection (RS), direction-based crossover (DBX), and the dynamic random mutation (DRM), to mimic a specific evolutionary process that has a parallel-structured inner loop. A variety of benchmark constrained optimization problems (COPs) are used to evaluate the effectiveness and the applicability of the proposed RCGA. Besides, some existing state-of-the-art optimization algorithms in the same category of the proposed algorithm are considered and utilized as a rigorous base of performance evaluation. Extensive comparison results reveal that the proposed RCGA is superior to most of the comparison algorithms in providing a much faster convergence speed as well as a better solution accuracy, especially for problems subject to stringent equality constraints. Finally, as a specific application, the proposed RCGA is applied to optimize the GaAs film growth of a horizontal metal-organic chemical vapor deposition reactor. Simulation studies have confirmed the superior performance of the proposed RCGA in solving COPs.



## 1. Introduction

Owing to the powerful capability of solving real-world optimization problems, the real-coded genetic algorithm (RCGA) is one of the most effective and commonly used evolutionary algorithm (EA), and many successful applications using RCGAs in diversified fields have been reported recently [1–5]. By mimicking the biological world of natural selection and survival of the fittest, the RCGAs are essentially a kind of population-based stochastic search schemes that implement the selection, crossover, and mutation operators in a series framework. To further improve the solution efficiency of RCGAs, many achievements and remarkable efforts have been completed and reported in the past few decades. According to mechanisms and techniques used, the previous developments and attempts made for RCGAs can be classified into the following categories: (1) the determination of an optimal population size [6–9]; (2) the initialization of population [10–12]; (3) the automatic adjustment of operator parameters [13–15]; (4) the control of population diversity [16–19]; (5) the improvement of existing crossover operators [20–22]; (5) the development of new crossover schemes [23–32]; (6) the investigation of novel evolutionary strategies [33–35]; and (7) the hybrid use of evolutionary operators [36–39]. The above summary shows that much emphasis has been placed to improve the crossover operations. A reason for this is that the crossover operators provide a featured ability to generate new candidate solutions by recombining parent's genes. More recently, Chuang et al. [40] completed a full survey on the techniques used to advance crossover operators [23–26,28–32,34] and based on the survey they found that the line segment connection and distribution analysis of parents are the two most commonly used schemes to develop new crossover operators. They also pointed out that the crossover operators developed based on these two techniques may encounter difficulties when faced with stringent optimization problems. More precisely, these crossover operations might not effectively generate potential offspring in some ambiguous regions as the size of population is relatively small compared to the whole searching space or the distribution of the initial population is not uniformly distributed over the entire admissible domain [25]. Besides, the designed crossover operators could hardly locate the global optimum once the true solution lies close to or on the boundaries of the feasible search space [27].

* Corresponding author at: Center of General Education, National Quemoy University, No. 1, University Rd., Jinning Township, Kinmen 892, Taiwan.
Tel.: +886 82 313901; fax: +886 82 313903.
*E-mail address:* ctchen@nqu.edu.tw (C.-T. Chen).

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To overcome these difficulties, this paper develops an RCGA that implements a specially designed direction-based crossover operator (DBX) to guide the crossover along a potential direction that is able to significantly improve the fitness, even though the COP is subject to the above-mentioned stringent conditions. By making use of the relative fitness information derived from the objective function and the current status about the constraint violations, the DBX crossover operator essentially explores $2^n - 1$ possible searching directions to generate better offspring, where $n$ is the number of genes. Besides, unlike conventional crossover operators that use a fixed value or a random number to control the step size of a crossover, the DBX adaptively adjust the crossover's step-size by using the fitness distance calculated from the chosen paired-parents and the maximum fitness difference measured in the current population. In addition, the proposed RCGA distinctively constitutes a coordinator in the parallel inner-loop to organize the operation of the DBX and DRM [41] in order to provide a higher possibility of locating the true global optimum under stringent search conditions. Furthermore, a parameter selection guideline for the proposed RCGA is provided and the comprehensive studies are performed to investigate the effectiveness of each operator in the algorithm structure. The effectiveness and applicability of the proposed RCGA in solving COPs are demonstrated through a variety of benchmark COPs, followed by the comparisons with some existing state-of-the-art EAs on the CEC2006 testbed. The related statistical data, such as the success rate, performance index, performance rankings and the complexity index of the proposed RCGA, are reported. The remainder of this paper is organized as follows. In Section 2, the COPs under study are described and a static constraint-handling technique to work with the proposed RCGA is introduced. Subsequently, the algorithm structure and the operator functions specially developed for the RCGA are characterized and analyzed. In Section 3, the effectiveness of these evolution operators implemented in the proposed RCGA is systematically investigated and demonstrated. Thereafter, Section 4 performs comparative studies on a variety of COPs, where the benchmark testbed, experimental setup, and the comparison algorithms are presented and discussed. As a specific application study, in Section 5, the proposed RCGA is applied to determine an optimal set of operating condition for a horizontal metal-organic chemical vapor deposition (MOCVD) reactor to maximize its film-growth performance. Finally, in Section 6, concluding remarks are given for the proposed RCGA in solving COPs.

## 2. The development of the proposed RCGA for constrained optimization

### 2.1. Problem statement and a static penalty function

The COP considered in this paper is formulated as follows:

$$\begin{aligned} &\text{minimize} \quad f(\mathbf{x}), \quad \mathbf{x} = [x_1, x_2, \ldots, x_n] \in S \\ &\text{subject to} \quad g_p(\mathbf{x}) \le 0, \quad p = 1, 2, \ldots, P \\ &\qquad\qquad\quad h_q(\mathbf{x}) = 0, \quad q = 1, 2, \ldots, Q \end{aligned} \tag{1}$$

where $f(\mathbf{x})$ is the objective function to be minimized, $g_p(\mathbf{x}) \le 0$ denotes the $p$th inequality constraint, $h_q(\mathbf{x}) = 0$ represents the $q$th equality constraint, $\mathbf{x}$ is the vector of the $n$-dimensional decision variables, and $S$ is the admissible search space restricted to the parametric constraint bounds as follows:

$$x_{j,\min} \le x_j \le x_{j,\max}, \quad j = 1, 2, \ldots, n \tag{2}$$

where $x_{j,\min}$ and $x_{j,\max}$ stand for the lower and upper bounds of $x_j$, respectively. The feasible region $F \subseteq S$ formed by $g_p(\mathbf{x}) \le 0$ and $h_q(\mathbf{x}) = 0$ is expressed as

$$F = \left\{ x \in S | g_p(\mathbf{x}) \le 0,\ p = 1, 2, \ldots, P;\ h_q(\mathbf{x}) = 0,\ q = 1, 2, \ldots, Q \right\} \tag{3}$$

Let $\mathbf{x}^*$ be the optimal solution for the COP, then we have $\forall \mathbf{x} \in F \subseteq S : f(\mathbf{x}^*) \le f(\mathbf{x})$. Besides, the inequality constraint is called the *active* constraint if $g_p(\mathbf{x}^*) = 0$. In this concept, all the equality constraints $h_q(\mathbf{x}) = 0$, $q = 1, 2, \ldots, Q$, are active at $\mathbf{x}^*$. To evaluate the satisfaction of the constraints and to judge the objective function value simultaneously, this paper applies a static penalty function (SPF) to work with the proposed RCGA, which is given by [42]

$$\tilde{f}(\mathbf{x}) = f(\mathbf{x}) + C_I \sum_{p=1}^{P} g_p^+(\mathbf{x}) + C_E \sum_{q=1}^{Q} h_q^+(\mathbf{x})^2 \tag{4}$$

where $C_I$ and $C_E$ are the user-specified penalty factors, and the indication functions $g_p^+(\mathbf{x})$ and $h_q^+(\mathbf{x})$, which are used to account for the violations of the inequality and equality constraint, are, respectively, defined by

$$g_p^+(\mathbf{x}) = \max\{g_p(\mathbf{x}), 0\}, \quad p = 1, 2, \ldots, P \tag{5}$$

$$h_q^+(\mathbf{x}) = \max\{|h_q(\mathbf{x})| - \xi, 0\}, \quad q = 1, 2, \ldots, Q \tag{6}$$

In the above equations, the max-operator {} returns zero as the solution $\mathbf{x}$ belongs to $F$; otherwise, a positive value will be produced to penalize the infeasible solution. Note that, to significantly amplify the penalty function for infeasible solutions, the penalty factors $C_I$ and $C_E$ are normally set to positive constants that are large enough. Besides, in Eq. (6), the parameter $\xi$ is used to ease the difficulty of dealing with equality constraints. Usually, a small positive value of $\xi$ is required to meet the required accuracy level of equality constraints, and its suitable value needs to be carefully chosen because it dramatically controls the allowable room on which an evolutionary algorithm can work properly [42,43]. In addition, because there are no fixed penalty factors that are suitable for every COP, one usually needs to conclude from numerical experiments a proper set of penalty factors for a specific application [42–45]. It should be mentioned here that, although this paper applies the SPF, Eq. (4), as the performance (fitness) index to work with the proposed RCGA, other types of constraint-handling techniques [43–50] can also be used in the same framework.

### 2.2. The algorithm structure and operators of the proposed RCGA

#### 2.2.1. The algorithm structure and the RCGA notations

Fig. 1 schematically depicts the configuration of the proposed RCGA for solving the COP in Eq. (1), where three specially designed evolutionary operators, named RS, DBX, and DRM, are integrated as a whole to mimic a specific evolutionary process. Before presenting these novel evolutionary operators and the algorithm structure, the underlying design concept and notations are briefly introduced as follows. Let $\boldsymbol{\theta} = [x_1, x_2, \ldots, x_n]$ be a solution termed as chromosome in the sense of the RCGA. Each $x_j$, where $j \in \bar{n}$ and $\bar{n} = \{1, 2, \ldots, n\}$, in the chromosome is called a gene and represented as a real number. The admissible parameter space for $\boldsymbol{\theta}$ is defined as follows:

$$S = \{\boldsymbol{\theta} \in \Re^n | x_{1,\min} \le x_1 \le x_{1,\max}, x_{2,\min} \le x_2 \le x_{2,\max}, \cdots, x_{n,\min} \le x_n \le x_{n,\max}\}. \tag{7}$$

Note that an initial population with $N$ chromosomes in $S$ is randomly created to start the RCGA. In what follows, the operational procedure and operators associated with the proposed RCGA will be introduced and investigated.