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Stochastic numerical solver for nanofluidic problems containing multi-walled carbon nanotubes

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ABSTRACT

In the present study, a new soft computing framework is developed for solving nanofluidic problems based on fluid flow and heat transfer of multi-walled carbon nanotube (MWCNT) along a flat plate with Navier slip boundary with the help of artificial neural networks (ANNs), Genetic Algorithms (GAs), Interior-Point Algorithm (IPA), and hybridized approach GA-IPA. Original PDEs associated with the problem are transformed into system of nonlinear ODEs using similarity transformation. Mathematical model of transformed system is constructed by exploiting the strength of universal function approximation ability of ANNs and an unsupervised error function is formulated for the system in a least mean square sense. Learning of the design variable of the networks is carried out with GAs supported with IPA for rapid local convergence. The design scheme is applied to solve number of variants by taking water, engine oil, and kerosene oil as a base fluids mixed with different concentrations of MWCNTs. The reliability and effectiveness of the design scheme is measured with the help of results of statistical analysis based on sufficient large number of independent runs of the algorithms rather than single successful run. The comparative studies of the proposed solution are made with standard numerical results in order to establish the correctness of the given scheme.

1. Introduction

Heat transfer fluids (HTFs) having appropriate thermal conductivity properties are considered to be fundamental in microelectronics cooling, chemical production, refrigerator and air-conditioning, transportation, and many other applications [1–3]. Increase in the thermal conductivity of these fluids is desired to improve heat transfer rate that is achieved usually with the additions of nanoparticles. Fluid flow and heat transfer problems involving nanoparticles have been investigated by many researchers due to their significant impact on industrial applications, including extrusion of plastic, paper production, metal spinning, wire drawing, glass blowing, hot rolling, manufacture of rubber sheet, polymer engineering, cooling of metallic sheets, and crystal growing [3–8].

In the class of nanofluids, the use of cylindrical multi-walled carbon nanotubes (MWCNTs) is more often due to their unique characteristic. The discovery of MWCNTs is associated with Japanese physicist S. Iijima during 1990s, in his work related to the insoluble soot of arc-burned graphite rods [9]. Since then, these carbon nanotubes (CNTs) gained utmost attention as a valuable nanomaterial with extensive applications [10,11]. CNT is given such a specific name due to their tube type shape and formulation with carbon. The MWCNTs are specific type of CNTs that are based on collection of numerous cylinders, which are separated by 0.35 nm [12]. These nanoparticles have wide range of characteristics, such as smooth structure plane on nanoscale, light weight, extraordinary strength to resist a temperature upto 2000 °C, and are considered to be one of the strongest and stiffest material discovered due to their properties of high tensile strength and elastic modulus [13–15]. Thermal and electrical conductivity properties of MWCNTs are based on reinforced composites [16,17], which

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are exceptional and that enable their facile and nondestructive characterization. MWCNTs arise in many studies such as enhancement of electrochemically solid-phase microextraction based on molecularly imprinted MWCNTs composite coating [18], use the roots of kiwi fruit samples for development of molecularly imprinted polymers based on MWCNTs [19], formulation of chitosan and cellulose nanocrystals using layer-by-layer assembled films of MWCNTs [20], hybrid nonoparticles consist of gold, and MWCNTs are used for amplified electrochemical detection of protein kinase activity [21], reliable extinction coefficient of aqueous nanofluids based on MWCNTs [22] and MWCNTs composite electrode used in rechargeable batteries [23], etc. Due to their utmost importance of MWCNTs in many fields, research community attracted to develop analytical and numerical solvers for solving fluid mechanics problems involving these nanoparticles mixed with different type base fluids [24–26]. However, beside the well-established strength of stochastic numerical solver, these artificial intelligence procedures have not been exploited for solving nanofluid dynamics problems involving MWCNTs.

The stochastic numerical solvers based on feed-forward artificial neural networks (FF-ANNs) optimized with evolutionary and swarm intelligence algorithms, which have been applied immensely to solve a variety of linear/nonlinear initial value problems (IVPs) and boundary value problems (BVPs) of differential equations [27,28]. For example, these solvers are used for BVPs of pantograph functional differential equation [29], nonlinear singular systems associated with Emden–Flower equations [30], solving single variable nonlinear algebraic, and transcendental equations [31], strong nonlinear problem of first Painlevé transcendent [32–35], plasma physics problem associated with Toresch's BVPs [35,36], computational fluid mechanics problems models with nonlinear Jeffery–Hamel flow equation in the presence of high magnetic field [37,38], nonlinear Riccati system represented with fractional differential equations (FDEs) [39,40], and homogeneous and nonhomogeneous IVPs of Bagley–Torvik FDEs [41]. Recently, BVPs of thin film flow of third grade fluid [42], fuel ignition model of combustion theory given by nonlinear one-dimensional Bratu's BVPs [43,44], and transformed problems of two-dimensional nonlinear Bratu's equations are other illustrative potential application of these schemes [45–47]. The capability of ANNs are also applied extensively in nanotechnology problems of computational fluid dynamics, such as determination of viscosity in water-TiO_2 nanofluid [49], prediction of heat transfer due to presence of copper—water nanofluid [50], prediction of unsteady mixed convection over circular cylinder in the presence of nanofluid [51], analysis of laminar mixed convection in an inclined square lid-driven cavity with a nanofluid [52], and prediction of thermal gradient in an air channel with presence of electrostatic field [53]. Additionally a good source of reference for ANNs applications to nanotechnology can be seen in book [54]. In all these applications soft computing techniques utilized supervised neural network optimized with local search methodologies for solving governing mathematical models for nanofluidic problems involving nanoparticles based on copper, aluminum gold, silver, etc but not a single study is available in which MWCNTs based nanofluidic problems solved by ANNs. MWCNTs based nanofluidic problems look a promising area to be explored with soft computing approaches. These are motivation factor for authors to investigate in stochastic techniques and design an alternate, accurate, and reliable computing platform for fluid mechanics problems containing MWCNTs.

In this article, partial differential equations (PDEs) representing fluid flow and heat transfer problem in case of flat plate with Navier slip boundary conditions involving MWCNTs are transformed into system that consists of two nonlinear ODEs of order upto three and these transformed equations are solved by using FF-ANNs, GAs, and IPAs. The FF-ANNs models of differential equations are used to construct the fitness function and learning of weights of these networks is carried out with GAs, as a global search operator, hybrid with IPA for viable local convergence. The proposed scheme is tested on number of scenarios for nanofluid dynamics by taking different base fluids and concentrations of multi-walled CNTs. The comparative study of the proposed solution is made with the result of fully explicit Runge–Kutta state of art numerical solvers in MATHEMATICA software package.

2. System model: problem formulation

Considering a two-dimensional flow over a flat plate with heat transfer involving nanofluids that consist of water, kerosene oil (KO), or engine oil (EO) as a base fluid mixed with different concentrations of MWCNTs. Fluid flow is considered to be laminar, steady, and incompressible so that it has constant density. Additionally, uniform heat flux is assumed on a plate surface. Boundary layer problem for these scenarios can be written with governing mathematical relations base on PDEs as: [55]

$$\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y} \tag{1}$$

$$u\left(\frac{\partial u}{\partial x}\right) + v\left(\frac{\partial v}{\partial y}\right) = \nu_{nf}\left(\frac{\partial^2 u}{\partial y^2}\right)$$

$$u\left(\frac{\partial T}{\partial x}\right) + v\left(\frac{\partial T}{\partial y}\right) = \alpha_{nf}\left(\frac{\partial^2 T}{\partial y^2}\right)$$

Here, u and v are representing the velocity profile of the fluid along the x-axis and y-axis, respectively, T represents temperature, ν_{nf} is the effective kinematic viscosity, while α_{nf} is the thermal diffusivity. The general characteristic of nanofluids may be given in terms of the properties of the base fluid, and solid volume fraction of MWCNTs, and given as follows:

$$\begin{cases} \rho_{nf} = (1-\phi)\rho_f + \phi\rho_{MWCNT}, \\ \mu_{nf} = \mu_f(1-\phi)^{-2.5} \\ (\rho c_p)_{nf} = \phi(\rho c_p)_{MWCNT} + (1-\phi)(\rho c_p)_f \\ \nu_{nf} = \dfrac{\mu_{nf}}{\rho_{nf}}, \quad \alpha_{nf} = \dfrac{k_{nf}}{(\rho c_p)_{nf}} \end{cases} \tag{2}$$