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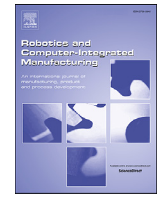

Kinematic error modeling and identification of the over-constrained parallel kinematic machine

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ABSTRACT

Kinematic calibration has proved to be an effective method in improving the kinematic accuracy of the parallel kinematic machine (PKM). Though the PKM's kinematic calibration method has achieved substantial progress, its application is limited and can be only utilized in non-redundant PKM. Over-constrained PKM parts are forced to deform under redundant constraint forces to satisfy the assembly condition, which does not meet the assumptions made in the present PKM's kinematic calibration method in which all the parts are rigid. To deal with this issue, the present study investigates the kinematic error modeling and identification of the over-constrained PKM considering the deformations of the parts. A 2-DOF over-constrained PKM is taken as the object of study and its kinematic error model is derived based on the analyses of the link's deformations and the machine's static equilibrium condition. The method of least squares and the Regularization method are adopted to identify the kinematic errors of the over-constrained PKM. Finally, simulation analyses are carried out to verify the feasibility and effectiveness of the proposed method.

1. Introduction

The parallel kinematic machine (PKM) has higher stiffness, better dynamic performance, larger payload capacity, and higher modularization degree as compared with the serial machine [1–3]. As a result, PKM has garnered great attention for its wide applicability in industrial process. In particular, PKM has been successfully applied in the application of pick-and-place operations due to its high-speed performance [4,5]. However, PKM has not achieved much success in high-precision applications [6,7] as it is difficult to guarantee the PKM's kinematic accuracy. Numerous passive joints in the PKM as well as its kinematic chains form several complex closed-loop structures, which complicate its measurements and improvements towards better manufacturing and assembly accuracy [8–10]. Additionally, direct and timely measurements of the PKM's terminal platform pose are almost impossible in practice given that the PKM's kinematic accuracy cannot be compensated by using closed-loop control as the serial machines. Improvements in the PKM's kinematic accuracy have proved challenging and are crucial for its practical application.

Two main methods, namely accuracy analysis and synthesis [11–15] and kinematic calibration [16–20], can be utilized to improve the PKM's kinematic accuracy. Generally, accuracy analysis and synthesis is carried out in the design process of the machine. On the basis of the PKM's kinematic error model, the influence of the geometric errors on the PKM's kinematic accuracy is analyzed to determine their tolerance zones according to the accuracy design requirement. Merlet [11] discussed the dimensional synthesis of the *n*-DOF PKM with respect to the guaranteed given accuracy over a specific workspace. An interval analysis-based approach was proposed to solve this issue with the advantages of not only providing one solution but also providing a continuous set considers the manufacturing errors. Tang [12] examined the accuracy synthesis of a multi-level hybrid positioning mechanism for the feed support system in the Five-hundred-meter Aperture Spherical Radio Telescope (FAST) project. The terminal error boundary across the entire workspace was acquired by the vector set theory and a linear algebra method, and the terminal accuracy synthesis was conducted by a nonlinear optimization algorithm. Li [13] investigated the number-theoretic method with Sobol point to improve the accuracy design efficiency of a 6-DOF docking mechanism. The proposed method can attain the same precision with only a tenth of the required sample size as compared with the traditional Monte Carlo method. Though accuracy analysis and synthesis guides the design, fabrication, and assembly of the PKM, high determined manufacturing and assembly accuracy vales may significantly increase the PKM's fabrication cost or deter its applicability in practice.

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In contrary, kinematic calibration is implemented following the fabrication and assembly of the machine. According to the measured motion information, the manufacturing and assembly errors of the PKM are identified resorting to the established kinematic error model and the identification algorithm, of which the kinematic errors can be compensated by the PKM's driving system. Wu [6] presented highly accurate and effective approach for the kinematic calibration of a 5-DOF hexapod machine tool with a laser tracker. An optimal model combined with an arithmetic mean method was proposed to identify the kinematic parameters, which effectively reduced the PKM's kinematic errors caused by the inaccurate motions of the joints. Huang [7] proposed a new measurement scheme to measure the pose of a 3-PRS-XY hybrid machine tool and explored solutions for the ill-posed problem in the kinematic error identification based on the Regularization method. Using kinematic error compensation, the test illustrated a less than 0.05 mm improvement in the RTCP accuracy of the machine tool. Tian [16] presented a simple and effective approach for the kinematic calibration of a 3-DOF spindle head using a double ball bar, of which the results showed a significant improvement in the compensable pose accuracy. Sun [17] proposed the laser tracker based kinematic calibration of a 3-DOF rotational PKM, which was implemented in four steps. The experimental tests exhibited a > 53.4% improvement in the PKM pose accuracy by error compensation. The kinematic calibration can prominently and directly improve the machine's kinematic accuracy without increasing the manufacturing cost as compared to accuracy analysis and synthesis and has thus been a key technique in improving the PKM's kinematic accuracy.

In practice, a redundant constraint is generally introduced into the PKM to enhance its stiffness and operational stability, thus extending the applicability of the over-constrained PKM [21–24]. To improve the kinematic accuracy of the over-constrained PKM, the kinematic calibration process is also necessary to be carried out to identify and compensate for its manufacturing and assembly errors. Unlike the non-redundant PKM, redundant constraint forces occur in the over-constrained PKM so along as manufacturing and assembly errors are present due to the over-constrained feature of the machine. The parts of the over-constrained PKM are then forced to deform to satisfy the assembly condition of the machine. The deformations of the parts are coupled with the structural errors and vary with the PKM's pose, which will affect the machine's kinematic accuracy. Therefore, the kinematic error modeling and identification with the consideration of the parts deformations are the key issues in the kinematic calibration process of the over-constrained PKM. However, current kinematic calibration methods assume PKM parts to be rigid in the kinematic error model [29–31] and not consider their deformations during the kinematic calibration process. At present, deformations in the PKM parts are considered in its stiffness modeling or dynamic characteristic analysis. Hu [25] derived the formulae for solving the elastic deformation and the compliance matrix of the active legs of a 2(PS+SPR+SPU) serial-parallel manipulator to analyze its total stiffness. Wang [26] investigated the running accuracy of a 3-RRR PKM considering the presence of link deformations during the motion process. Jiang [27,28] established the dynamic error model for redundantly actuated and kinematic PKMs by analyzing the deformations of kinematic chains, wherein redundant forces are optimized to improve the precision of the machines.

Though PKM kinematic calibration method has substantially progressed and is not far from practical use, its application is limited and is not appropriate for the over-constrained PKM. Therefore, the present study investigates the kinematic error modeling and identification of the over-constrained PKM with the consideration of the deformations of the parts. The remainder of this paper is organized as follows. The systematic process for the kinematic error modeling of the non-redundant PKM is first introduced in Section 2. A 2-DOF planar over-constrained PKM is taken as the object of study in Section 3. Taking the links' deformations into consideration, the kinematic error model of this PKM is then derived in Section 4. Kinematic error identification of the over-constrained PKM is discussed in Section 5. Finally, the conclusions of this paper are presented.

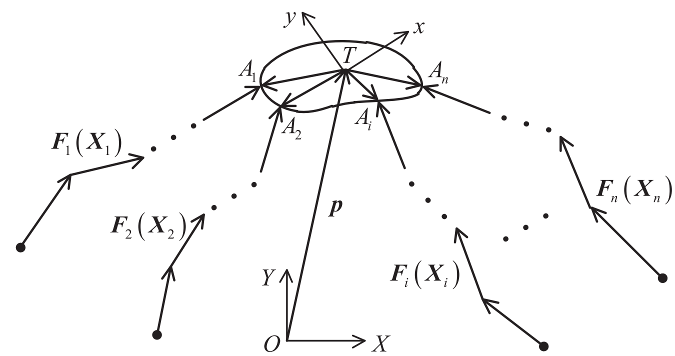

Fig. 1. Kinematic vector loops of the PKM.

2. Systematic process for the kinematic error modeling of the non-redundant PKM

The kinematic error model of the PKM, which establishes the relationship between the geometric errors and the kinematic accuracy of the terminal platform, is the basis of the accuracy analysis and kinematic calibration.

The PKM's geometric errors are miniscule as compared to its geometric parameters and can be viewed as perturbation terms. Therefore, the PKM's kinematic error model can be derived from the perturbation of the ideal kinematic model. The PKM's kinematic model is first established and the schematic of the kinematic vector loops is shown in Fig. 1. A global coordinate frame {*O-XYZ*} is located at the fixed platform and a moving coordinate frame {*T-xyz*} is attached to the terminal platform.

The pose descriptions of the PKM parts are necessary for the kinematic model. Although all the PKM part pose descriptions are similar in essence, the corresponding descriptions for the perturbation-obtained geometric errors through perturbation are more distinct. The part's geometric error descriptions, which include the manufacturing and assembly errors, are based on clear physical meanings to convenience the accuracy analysis and kinematic calibration. The PKM part manufacturing errors are described as the positional errors in its local coordinate frame, whereas the assembly errors are described as the pose errors of the local coordinate frame. Therefore, local coordinate frames are established and attached to each part of the PKM to describe their poses and geometric errors.

According to Fig. 1, the closed-loop constraint equation of the *i*th kinematic vector loop can be expressed as

$$\boldsymbol{p} + \boldsymbol{R}\boldsymbol{m}_i = \boldsymbol{F}_i(\boldsymbol{X}_i) \tag{1}$$

where $\boldsymbol{p}$ is the position vector of origin *T* in global coordinate frame {*O*}, $\boldsymbol{m}_i$ is the terminal position vector of the *i*th kinematic chain in coordinate frame {*T*}, $\boldsymbol{R}$ is the rotation matrix of coordinate frame {*T*} with respect to coordinate frame {*O*}, $\boldsymbol{X}_i$ is the geometric parameters of the *i*th kinematic chain, and $\boldsymbol{F}_i$ is the kinematic mapping function.

Considering the kinematic errors of the PKM, the kinematic error equation of the *i*th kinematic chain can be obtained according to Eq. (1) as

$$\boldsymbol{p} + \delta\boldsymbol{p} + (\boldsymbol{R} + \delta\boldsymbol{R})(\boldsymbol{m}_i + \delta\boldsymbol{m}_i) = \boldsymbol{F}_i(\boldsymbol{X}_i) + \boldsymbol{J}_i\delta\boldsymbol{X}_i \tag{2}$$

Subtracting Eq. (1) from Eq. (2) and ignoring the higher order items yields

$$\delta\boldsymbol{p} + \delta\boldsymbol{R}\boldsymbol{m}_i + \boldsymbol{R}\delta\boldsymbol{m}_i = \boldsymbol{J}_i\delta\boldsymbol{X}_i \tag{3}$$

In the kinematic chain error model, there might be redundant errors which are errors that do not have influences or are non-independent influences on the PKM's kinematic accuracy. Redundant errors affect the