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Computationally efficient dynamic modeling of robot manipulators with multiple flexible-links using acceleration-based discrete time transfer matrix method

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ABSTRACT

This paper presents a novel and computationally efficient modeling method for the dynamics of flexible-link robot manipulators. In this method, a robot manipulator is decomposed into components/elements. The component/element dynamics is established using Newton–Euler equations, and then is linearized based on the acceleration-based state vector. The transfer matrices for each type of components/elements are developed, and used to establish the system equations of a flexible robot manipulator by concatenating the state vector from the base to the end-effector. With this strategy, the size of the final system dynamic equations does not increase with the number of joints or the number of link beam elements that each link is decomposed. The developed method intends to avoid the traditional computation of the global system dynamic equations that usually have large size for flexible robot manipulators, and only involves calculating and transferring component/element dynamic equations that have small size. The numerical simulations and experimental testing of flexible-link manipulators are conducted to validate the proposed methodologies.

1. Introduction

Traditional industrial robot manipulators are built to be massive in order to increase stiffness, and therefore move at speeds much lower than the fundamental natural frequency of the system due to the limitations in the joint actuator output torque. The practical solution to this problem is to design and construct light weight manipulators, which are capable of moving swiftly. In contrast to the rigid manipulators, light weight manipulators offer advantages such as higher speed, better energy efficiency, improved mobility, and higher payload-to-arm weight ratio. However, at high operational speed and acceleration, inertial forces of moving components become quite large, leading to significant deformation in the light links, and generating unwanted vibration. Hence, elastic vibrations of light weight links must be taken into account in the dynamic modeling, design, and control of the robot manipulators.

In the past decades, significant efforts have been made into the investigation of dynamic modeling of manipulators with flexible components [1–5]. Different discretization techniques, such as the finite element method (FEM) [6–13], the assumed mode method (AMM) [14–18], and the lumped parameter method (LPM) [19–22], have been reported extensively for modeling the flexible dynamics of robot manipulators. However, the matrix size of global dynamic model of a robot manipulator increases with the number of the Degrees of Freedom (DOF), and therefore heavy computation of dynamic modeling is still a big concern in terms of real-time control.

An alternative to FEM, AMM and LPM, the transfer matrix method (TMM) can be used to model linear and continuous elements without discretization [23–25]. However when the traditional TMM method is applied to multiple DOF mechanisms and manipulators, the global dynamics still needs to be established using Lagrange's equation or Hamilton's principle. The number of the global dynamic equations increases with the DOF of the system. Using the integration procedure, the discrete-time transfer matrix method (DT-TMM) was presented to perform the dynamic analysis of a large system that consists of a large number of subsystems, each of which is a simple dynamic element [26]. The DT-TMM was further developed to model multi-body system dynamics using linearization and integral schemes [27–29]. With the DT-TMM, the global dynamic equations using the traditional ways are avoided, and the matrix size of the dynamic equations does not increase with the DOF of multi-body systems. Therefore, the computation efficiency can be significantly improved.

In this work, the DT-TMM is extended to the dynamic modeling of light weight robot manipulators considering the link flexibility. Firstly, the basic principle and procedure of the DT-TMM are addressed with an overview in Section 2. Secondly, the detailed methodology is

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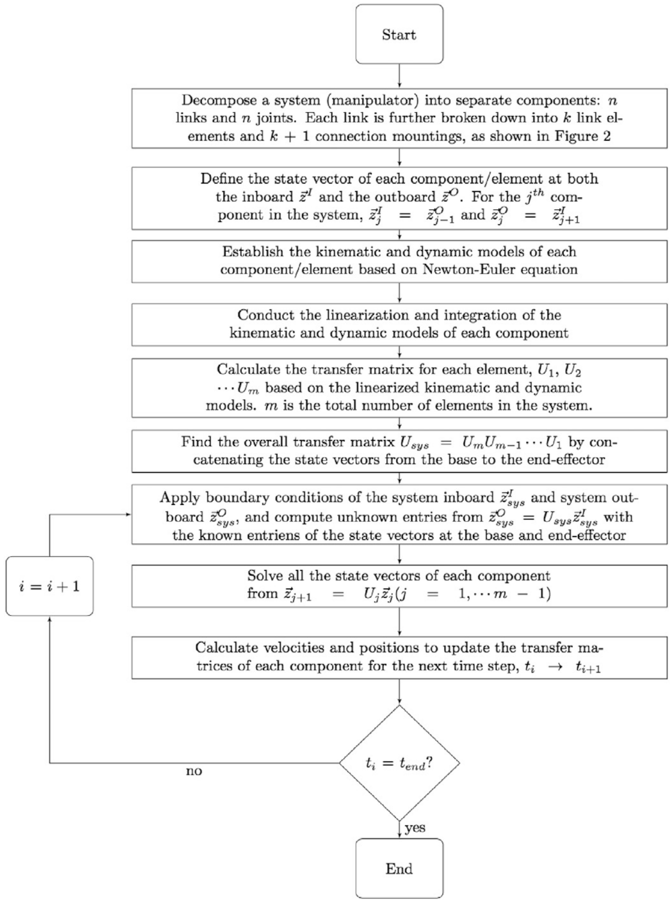

Fig. 1. Flow chart of the algorithm for the transfer matrix method.

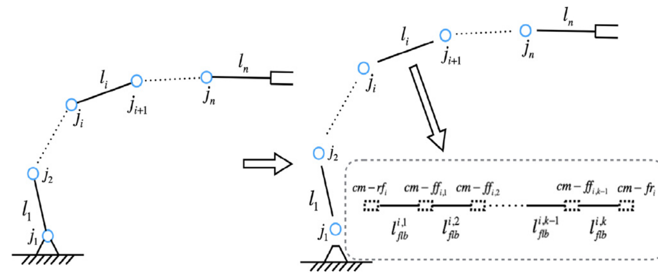

Fig. 2. This picture shows the process of decomposing a robot arm to elements/ components (n links: $l_1, l_2, \ldots, l_n$ and n joints: $j_1, j_2, \ldots, j_n$). Each link is further decomposed into k link beam elements and $k+1$ connection mountings.

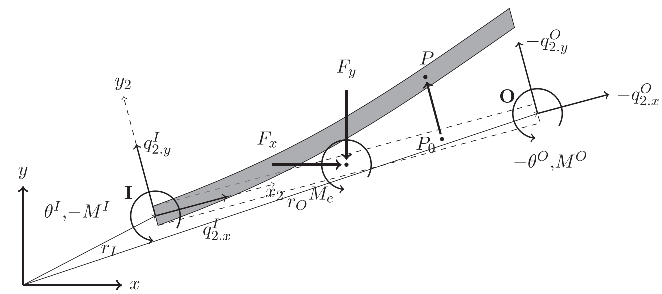

Fig. 3. State vectors and coordinate system. Deformation and length are measured in the local reference frame, denoted by subscript *2*. For simplicity, the external forces F_x and F_y, and moment M_e are applied at the center of gravity.

presented for the dynamic modeling of flexible-link robot manipulators. State vectors and transfer matrices are defined, and kinematic and dynamic models are established for each components/elements decomposed from a robot manipulator. The linearization is conducted for component/element kinematic and dynamic equations. The DT-TMM solving process is conducted with the formulation of component/element transfer matrices and generation of system dynamics by concatenating the state vectors from the base to the end-effector of the robot manipulator. Thirdly, the simulations and analyses of a 1-DOF robot manipulator with single flexible link and a 3-DOF robot manipulator with three flexible links are provided to demonstrate the developed DT-TMM. In addition, the experimental testing of a single flexible-link robot manipulator is performed to validate the modeling method.

2. Overview of DT-TMM: principles and procedures

The principle and procedure of the DT-TMM is illustrated with the schematic diagram in Fig. 1. With the proposed DT-TMM method, the global dynamics calculation using the traditional ways is avoided , and only the computation of element/component dynamics is conducted and transferred across the elements/components. The matrix size of the final system equations to solve dynamics does not increase with either the DOF of a robot manipulator or the number of link elements that are decomposed from flexible links. Apparently, the computation cost is significantly reduced.

The decomposing process of a 2-dimensional (2-D) robot manipulator is shown in Fig. 2. This 2-D robot manipulator is decomposed int of n joints and n links. Each links is further broken down into k link elements and $k+1$ connection mountings. $l^{i,j}_{flb}$ represents the jth$(j = 1, 2, \ldots, n-1)$ flexible link element of the ith link, $cm - ff_{i,j}$ the jth connection mounting with flexible inboard-flexible outboard of the ith link, $cm - rf_i$ the connection mounting between the ith link and the ith joint, and $cm - fr_i$ the connection mounting between the ith link and the $i+1$th joint. The dynamic analyses of these components/elements are detailed in following sections.

3. State vectors and transformation

In DT-TMM, the state vector is defined as a column vector that represents the internal forces (forces q_x, q_y, q_z, and moments M_x, M_y, M_z) and displacement (rigid motions x, y, z, θ_x, θ_y, θ_z and deformation modal coordinates $w^1, w^2, \ldots, w^n$) at a particular location within a system. For the acceleration-based integral scheme, the state vector is defined at any cross-section of a flexible link in 2-D space as

$$\vec{z} = [\overbrace{\ddot{x}, \ddot{y}, \ddot{\theta}}^{\text{acceleration}}, M, q_x, q_y, \overbrace{\ddot{w}^1, \ddot{w}^2, \ldots, \ddot{w}^n}^{\text{modal acceleration}}, 1]^T \tag{1}$$

In this work, the state vector is defined based on 2-D (planar) robot manipulators, and only the lateral deformation of flexible links is considered. The sign convention of elements in the state vector related to the reference coordinate system is illustrated in Fig. 3. A position coordinate or an orientation angle is defined as positive when it is in the positive direction of the coordinate axis. The inboard force or outboard moment applied on the element is positive if it is in the positive direction of the coordinate axis, and outboard force or inboard moment on the element is negative if it is in the positive direction of the coordinate axis [27]. The system inboard side is defined to be the base, and the system outboard side is defined to be the end-effector. The inboard side of a component/element is defined to be the side close to the base of a robot manipulator, and the outboard side of a component/element is defined to be the side close to the end-effector.

In the DT-TMM, a transfer matrix is employed to transform the state vector form one end of the component to the other end of the component. A transfer matrix is formulated based on the kinematic and dynamic equations of the component. For a robot manipulator, the dimension of the system transfer matrix $\mathbf{U}_{sys}$ depends on and matches the size