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Stiffness analysis of parallel manipulators with linear limbs by considering inertial wrench of moving links and constrained wrench

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ABSTRACT

A stiffness of the parallel manipulators with linear limbs is analyzed by considering the inertial wrench of the moving links and the constrained wrench. First, a formula is derived for solving the dynamic active and constrained wrenches, the inertial wrench of moving links based on the principle of virtual work. Second, the relationship between the elastic deformations of limbs and the dynamic active/constrained wrench and the inertial wrench of moving links are discovered and analyzed. Third, a unified stiffness model of parallel manipulators is established by considering the inertial wrench of moving links and the dynamic active/constrained wrench. Fourth, a unified formula is derived for solving the elastic deformations of the moving platform by considering the inertial wrench of the moving links and the dynamic active/constrained wrenches. Finally, an analytic numerical example of the 3SPR-type parallel manipulator is given for solving its stiffness and elastic deformation. The correctness of derived formulae of the stiffness and the elastic deformations are verified by the analytic numerical solutions.

1. Introduction

Parallel manipulator (PM) has their special merits and has been applied widely in some industries [6,20]. Stiffness is one of the important indices for evaluating the PM performances, particularly when the PM are used as the machine tools and the robot arm/legs, and higher stiffness allows higher machining speed with higher accuracy of the end-effector. However, it is challenging issue to establish a precision stiffness model of various PMs. In this aspect, Gosselin [6] discovered stiffness mapping relation of PMs. Portman et al. [20] studied dynamic collinear stiffness of Gough-Stewart platform without considering limbs' mass. Zhang et al. [28] proposed an elastodynamic model for optimal design and performance improvement of the parallel kinematic machine by counting natural frequencies in the workspace. Yuan et al. [27] analyzed static stiffness and the dynamic stiffness of cable-driven PMs as considering the effect of both cable mass and elasticity by identifying the robot natural frequencies. Yan et al. [26] analyzed the total deformation of specific parallelogram-type parallel manipulators (PMs) using a strain energy method considering the compliances of the mobile platform. Lum et al. [16] proposes to use a structural optimization approach to synthesize and optimize the topology, shape and size of the FPMs' sub-chains for achieving optimal dynamic and stiffness properties. Hoevenaars et al. (2011) presented a Jacobian-based stiffness analysis method for PMs with non-redundant legs based on screw theory. Klimchik et al. [10] studied a stiffness modeling for perfect and non-perfect PMs under internal and external loadings to compute and compensate the compliance errors. Kim et al. (2014) derived a leg stiffness matrix of PMs with serially connected legs by considering the effect of passive joints using reciprocal screws. Wu et al. [23,25,22,24] investigated the stiffness of several PMs and studied their kinematics and dynamics. In order to enhance the stiffness analysis of serial and PMs with passive joints, Pashkevich et al. [17,18] presented a non-linear stiffness model, and presented a stiffness modeling for over-constrained PMs with flexible links and compliant actuating joints based on a multidimensional lumped-parameter model. Aginaga et al. [1] proposed a methodology for calculating the stiffness matrix of 6- RUS PM. Li and Xu [12] studied the stiffness characteristics of a 3PUU PM by considering actuations and constraints. Here, (R, P, U, C, S) represent (revolute, prismatic, universal, cylinder, spherical) joints, respectively. Carbone and Ceccarelli [2] deduced the stiffness matrix of a hybrid PM and analyzed its stiffness performance. Cheng et al. [3] analyzed the stiffness characteristics of the 3CPS PM based on the principle of virtual work. Chi and Zhang [4] improved the system stiffness of the reconfigurable PM to locate the highest system stiffness, single and multi-objective optimizations. Enferadi et al. (2011) analyzed the stiffness of a 3-RRP

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Nomenclature

Symbol description

B, m	base and moving platform
PM	parallel manipulator
R, S, P	revolute, prismatic, spherical joints
$\boldsymbol{r}_i$	vector of the ith limb (i=1,..., n)
$\boldsymbol{\delta}_i$, r_i	the length of $\boldsymbol{r}_i$ and its the unit vector
$\boldsymbol{f}_{ci}$, $\mathbf{c}_i$	k constrained force and its unit vector
$\boldsymbol{t}_{ci}$,$\boldsymbol{\tau}_i$	n-k constrained torque and its unit vector
B_i, b_i	connection point of r_i at B and m
$\boldsymbol{E}_i$, $\boldsymbol{e}_i$	vector from O to B_i and vector from b_i to o
$\boldsymbol{d}_i$	the distance of o to $\boldsymbol{f}_{ci}$
$\boldsymbol{J}$	Jacobian matrix of PM
$\boldsymbol{\omega}_{gi}$, $\boldsymbol{a}_{gi}$, $\boldsymbol{\varepsilon}_{gi}$	angular velocity, acceleration, angular acceleration of r_i at its mass center
m_o, m_{gi}	mass of m and r_i
$\boldsymbol{I}_o$,$\boldsymbol{I}_{gi}$	inertial moment of m and r_i
{B}	coordinate system of B
{g_i}	coordinate system of r_i at g_i (g=p, q)
f_{ai}, t_{ai}	active force and torque
$\boldsymbol{f}_{ai}$, $\boldsymbol{t}_{ai}$	active force vector and torque vector
$\boldsymbol{f}_{ci}$, $\boldsymbol{t}_{ci}$	Constrained force and torque
$\boldsymbol{C}_g$	general flexibility matrix of n limbs in {B}.
$\boldsymbol{F}_a$	general input active wrench applied onto r_i,
$\boldsymbol{F}_c$	general constrained wrench exerted onto r_i
$\boldsymbol{F}_d$,$\boldsymbol{T}_d$	dynamic workload wrench applied onto m at o
$\boldsymbol{F}_g$, $\boldsymbol{T}_g$	general inertial wrench applied onto n limbs
$\boldsymbol{K}_g$	general stiffness matrix of n limbs
$\boldsymbol{f}_m$,$\boldsymbol{t}_m$	inertial wrench of m applied onto m at o
$\boldsymbol{f}_{pi}$, $\boldsymbol{t}_{pi}$	inertial wrench of piston rod in r_i in {B}
$\boldsymbol{f}_{qi}$, $\boldsymbol{t}_{qi}$	inertial wrench of cylinder in r_i in {B}
$\boldsymbol{G}$	gravity acceleration
e_i, $\boldsymbol{e}_i$	the distance from o to b_i and its vector.
$\boldsymbol{f}$, $\boldsymbol{t}$	central workload wrench applied onto m
$\boldsymbol{J}_{\omega i}$	mapped matrix from angular velocity of r_i to general velocity of m
$\boldsymbol{J}_{rgi}$	mapped matrix from translational velocity of r_i at g_i to general velocity of m
$\boldsymbol{J}_{gi}$	mapped matrix from general velocity of r_i at g_i to general velocity of m
$\boldsymbol{J}_g$	general mapped matrix from general velocity of n active limbs at g_i to general velocity of m
${}^{B}_{gi}\boldsymbol{R}$	rotational transform matrix from {g_i} to {B}
E_e, G	linear and rotational modular of elasticity
I, I_p	linear and rotational moment of inertia
d_{gi}, A_{gi}	diameter and cross section area of r_i
⊥, ||	perpendicular, parallel constraints

spherical PM based on strain energy and Castigliano's theorem. Ivanov and Corves [9] adopted a universal analytical method for stiffness-oriented design of flexure hinge-based PM. Pinto et al. [19] proposed a general methodology for obtaining static stiffness maps in lower mobility PMs. Sadjadian and Taghirad [21] studied kinematic modeling and singularity and stiffness of a 3-DoF redundant PM. Enferadi, Hoevenaarset, Kimal et al. proposed different stiffness analysis methods for PMs by neglecting inertial wrench of the moving links [7,8,11]. Lu et al. [13,15] studied the stiffness and the elastic deformation of several PMs based on the principle of virtual work and using computer-aided design variation geometry. Above studied have their merits and different focuses.

However, up to now, it has not been found to establish the stiffness model of PMs by considering the inertial wrench of the moving links and the dynamic active/constrained wrench and to discover its characteristics. In fact, the elastic deformations of PM generated by the inertial wrench of the mass of kinematic chain are quite large. Therefore, the inertial wrench of the moving links and the dynamic active/constrained wrench must be added into the stiffness model of PMs in order to solve the precision elastic deformations of PMs. For this reason, this paper focuses on the establishment of the stiffness model of PMs with linear limbs by considering the inertial wrench of moving links and the dynamic active/constrained wrench and to solve the precision elastic deformations of PMs.

2. General dynamics formula of PM

A dynamics formula is pre-condition for establishing the stiffness model of parallel manipulators (PMs) by considering the inertial wrench of moving links and the dynamic active/constrained wrench. A general PM with n linear limbs is shown in Fig. 1a. It is composed of a base B, a moving platform m, and n different linear limbs r_i (i=1, ..., n) for connecting B and m. Here, B includes n connecting points B_i and a central point O. m includes n connecting points b_i and a central point o.

Let $\boldsymbol{\delta}_i$ be the unit vector of r_i from B_i to b_i. Let e_i and $\boldsymbol{e}_i$ are the distance from o to b_i and its vector. Let {g_i}-$x_iy_iz_i$ be a coordinate system attached onto r_i at its mass center g_i (i=1, ..., n). Let {m}-xyz be a coordinate system attached onto m at o. Let {B}-XYZ be a coordinate system attached onto B at O. Let ○ be the vector operator, ○ may be one of (+, −, ×, •). Let $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$ be two vectors or two matrixes. If $\boldsymbol{U}$ can be solved from ($\boldsymbol{u}_1$ ○ $\boldsymbol{u}_2$) in {g_i}, $\boldsymbol{U}$ and its components in {g_i} and {B} can be represented as follows:

$$
\begin{aligned}
&{}^{gi}\boldsymbol{U} = {}^{gi}\boldsymbol{u}_1 \circ {}^{gi}\boldsymbol{u}_2 = {}^{gi}(\boldsymbol{u}_1 \circ \boldsymbol{u}_2),\\
&\boldsymbol{U} = \boldsymbol{u}_1 \circ \boldsymbol{u}_2 = (\boldsymbol{u}_1 \circ \boldsymbol{u}_2).
\end{aligned} \tag{1}
$$

Let $\boldsymbol{F}_a$ be the n×1 general input active wrench vector applied onto r_i, $\boldsymbol{F}_a$ includes u active forces and n-u active torques. Let $\boldsymbol{F}_c$ is the (6-n)×1 general constrained wrench vector exerted onto r_i; $\boldsymbol{F}_c$ includes k constrained forces and 6-n-k constrained torques. Let ($\boldsymbol{F}_d$, $\boldsymbol{T}_d$) be a dynamic workload wrench applied onto m at o. Let ($\boldsymbol{f}_{pi}$, $\boldsymbol{t}_{pi}$) be an inertial wrench of piston rod in r_i in {B}. Let ($\boldsymbol{f}_{qi}$, $\boldsymbol{t}_{qi}$) be an inertial wrench of cylinder in r_i in {B}.

A general dynamics formula of PM with n linear active limbs has been derived by [14] as below,

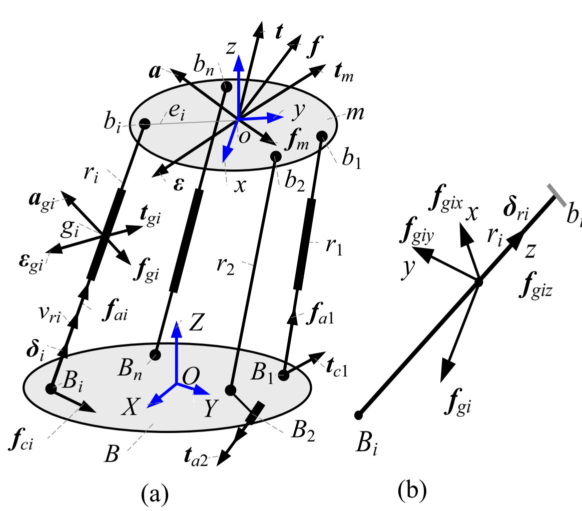

Fig. 1. A general PM (a), inertial force, its components of r_i (b).