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# A Bayesian approach to demand forecasting for new equipment programs

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ABSTRACT

Demand forecasting is a fundamental component in a range of industrial problems (e.g., inventory management, equipment maintenance). Forecasts are crucial to accurately estimating spare or replacement part demand to determine inventory stock levels. Estimating demand becomes challenging when parts experience intermittent demand/failures versus demand at more regular intervals or high quantities. In this paper, we develop a demand forecasting approach that utilizes Bayes' rule to improve the forecast accuracy of parts from new equipment programs where established demand patterns have not had sufficient time to develop. In these instances, the best information available tends to be "engineering estimates" based on like /similar parts or engineering projections. A case study is performed to validate the forecasting methodology. The validation compared the performance of the proposed Bayesian method and traditional forecasting methods for both forecast accuracy and overall inventory fill rate performance. The analysis showed that for specific situations the Bayesian-based forecasting approach more accurately predicts part demand, impacting part availability (fill rate) and inventory cost. This improved forecasting ability will enable managers to make better inventory investment decisions for new equipment programs.

## 1. Introduction

Demand forecasting is essential to inventory management. Inventory stock levels are dependent on forecasts of demand, and inaccurate estimation of spare part demand can lead to significant downtime costs. As a result, many systems incur large investments in spare parts inventories in an attempt to avoid 'stock outs'. A further complicating issue is that some spare parts experience intermittent demands, implying there are long periods of no demand followed by a series of demands in rapid succession. Intermittent demands create difficulties for traditional statistical demand forecasting methods.

The most common approach to forecast demand is to utilize statistical methods such as simple exponential smoothing. However, these approaches require observed demand data. When starting a new program, no historical information exists, which then requires the use of engineering estimates. No matter how confident one is in the estimate, all initial estimates of demand contain a considerable amount of uncertainty. Yet, new programs rely heavily on engineering estimates to determine optimal stock levels. For this reason, this paper explores the risk associated with the uncertainty of engineering estimates for spare parts in new programs and seeks to create a methodology that accounts for one's confidence in engineering estimates.

Another related question that needs to be explored is when an equipment program should transition from using engineering estimates to statistical approaches using observed demand. This issue is particularly important when equipment service level contracts are in place. In these cases, it is required to both minimize equipment program cost while still meeting agreed-upon service levels.

The current literature on demand forecasting for spare parts highlights a gap that exists between research and practice in the field of spare parts management [1,2]. Therefore, this paper focuses on developing a methodology that is easily implementable. The following section will first explore the related literature. Next, a Bayesian methodology is proposed to combine prior knowledge with observed data to obtain a new and improved estimate of demand. Finally, results of a case study are presented that illustrate the merits of the proposed approach.

## 2. Literature review

Over the past few decades, maintenance has become increasingly important for industrial environments resulting in growth in this research area. Effective maintenance is dependent on spare parts availability. Additionally, concern regarding intermittent and lumpy demand has been a focus of spare parts forecasting. Many reviews have noted a research-practice gap in the study of spare parts management.

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Adrodegari et al. [3] performed a critical review addressing spare parts inventory management and concluded that there are a limited number of papers that give a practitioner's view on how to apply proposed methods. The following review will examine three types of forecasting methods for spare parts management: judgmental, statistical, and Bayesian.

### 2.1. Judgemental forecasting

Judgmental forecasts are formed by expert opinion and are very common in practice when little to no historical data is available [4]. This type of forecasting is also frequently used as an adjustment method. Goodwin [5] pointed out that judgmental forecasting is used in three different instances 1) when there is a limited amount of historical demand as required by statistical methods; 2) when statistical models cannot exhibit effects of special events that may influence the future; 3) when modelers have a lack of understanding of statistical methods.

### 2.2. Statistical forecasting

Statistical forecasting techniques are typically used when historical data are available. There are numerous statistical forecasting methods discussed in the literature. However, this section focuses primarily on the literature related to spare parts forecasting using either time-series or bootstrapping approaches.

Time series forecasting methods find patterns in data to predict the future. Traditional time series methods such as moving average and simple exponential smoothing (SES) are often used in practice. SES is also largely used for inventory control system forecasting [6]. In order to cope with periods with zero demand, Croston [7] proposed a simple exponential smoothing technique that updates forecasts only in periods of demand. Hill et al. [8] pointed out that traditional time-series models can misjudge the functional relationship between independent and dependent variables, so they proposed a bootstrapping method. Willemain et al. [9] developed a heuristic to forecast intermittent demand for service parts using a bootstrapping approach.

### 2.3. Bayesian forecasting

A Bayesian approach utilizes available information and updates prior information (such as judgmental input) as observed demand occurs. The Bayesian paradigm has been used to overcome difficulties with limited demand data [10]. The objective of a Bayesian model is to evaluate posteriors to calculate an unknown statistic based on a likelihood function and a specified prior distribution. The likelihood function p(y|θ) represents the model for the observed data. The prior distribution p(θ) represents any prior knowledge the modeler knows about demand. The posterior p(θ|y) is the end goal that allows calculation of what the modeler believes the true demand rate is based on observed data (likelihood function) and prior knowledge (prior distribution). The prior, likelihood, and posterior are all related via Bayes' rule. Bayes' rule contains an integral in the denominator that is often intractable. However, the integral in the denominator of Bayes' rule can be avoided by using conjugate priors.

The Bayesian approach to forecasting demand for spare parts is not new in inventory control. In fact, Bayesian updating has been an active area of research in inventory control literature since the early 1950s. Scarf [11] was one of the first to propose Bayesian estimation in the context of a periodic review inventory model. However, most of this work evaluates inventory levels on an infinite horizon.

Several researchers have conducted comparative studies to support the use of Bayesian procedures. Aronis et al. [12] utilized a Bayesian approach to compare forecasts for electronic equipment spare parts demands versus other approaches. They found that their Bayesian approach resulted in lower stock levels at a 95% service level. Rahman and Sarker [13] explored a Bayesian approach to the forecasting of intermittent demand for seasonal products and found that their Bayesian model was very effective.

Most of the prior comparative studies use forecast accuracy or mean-value inventory optimization results to compare methods [1]. However, these results can lead to biased conclusions. Thus, this paper will evaluate the impact of forecasting method on "true" fill rate performance using simulation and determine whether these results agree with the forecast accuracy and inventory optimization analysis. The model developed will use Bayes' rule to forecast demand in a manner similar to Aronis et al. [12] since it will not change the inventory optimization model. The weights applied to the prior parameters give this research the unique ability to incorporate a confidence in the engineering estimates. This paper uniquely utilizes a Bayesian approach by 1) assuming the likelihood function is exponentially distributed (versus Poisson) and 2) developing a method to depict demand more accurately when zero demands have occurred.

## 3. Model formulation

A Bayesian forecasting approach immediately learns from observed demand and includes confidence in the engineering estimate. This section will apply Bayes' rule to demand forecasting. In this paper "demand" and "failure" are used interchangeably. In addition, it is assumed that the available demand data is aggregated over time and does not support the evaluation of a demand distribution that is dependent on operating hours. Therefore, the formulation of the Bayesian model requires the following assumptions:

1. **Likelihood Function:** Observed mean time between demands is exponentially distributed.
2. **Prior:** Engineering estimates (mean time between failures) are exponentially distributed. However, a Gamma function is used for the prior since the exponential distribution is a special case of the Gamma (α=1, β=mean) distribution.

Based on these assumptions, the posterior is formulated using Bayes' rule. The posterior is used to evaluate operating hours per demand (or mean time between failures). The unknown parameter of interest is $\lambda$, which is defined as operating hours per demand.

$$\textbf{Likelihood function (exponential):}\ L(n|\lambda)=\lambda^n e^{-\lambda}\sum_{i=1}^{n} x_i \tag{1}$$

$$\textbf{Prior (Gamma):}\ g(\lambda; r, v)=\frac{v^r\lambda^{r-1}e^{-v\lambda}}{\Gamma(r)}\propto\lambda^{r-1}e^{-v\lambda} \tag{2}$$

$v^r$ and $\Gamma(r)$ remain constant in respect to $\lambda$, so these parameters can be ignored when computing $\lambda$.

$$\textbf{Posterior (Gamma)}\ p(\lambda|\mathrm{n})=\frac{p(n|\lambda)\mathrm{p}(\lambda)}{p(n)}=\frac{p(n|\lambda)\mathrm{p}(\lambda)}{\int p(n|\lambda')\mathrm{p}(\lambda')\mathrm{d}\lambda'}\propto p(n|\lambda)\mathrm{p}(\lambda) \tag{3}$$

The integral (or partition function) in the denominator stays constant with respect to $\lambda$, so it can be ignored when computing $\lambda$. The posterior is computed as follows:

$$\begin{aligned} &p(\lambda|\mathrm{x,\ r,\ v})\propto p(n|\lambda)\mathrm{p}(\lambda)\\ &\propto(\lambda^n e^{-\lambda}\textstyle\sum_{i=1}^{n}x_i)(\lambda^{r-1}e^{-v\lambda})\\ &\propto\lambda^{n+r-1}e^{-(v+\sum_{i=1}^{n}x_i)\lambda} \end{aligned} \tag{4}$$

Eq. (4) is in the form of a Gamma distribution and is equivalent to Gamma $(r+n,\ v+\sum_{i=1}^{n}x_i)$. It is expressed as Gamma(r',v'). The full posterior equation follows:

$$p(\lambda|\mathrm{x,\ r,\ v})=\frac{v+\sum_{i=1}^{n}x_i{}^{r+v}\lambda^{r+n-1}e^{-(v+\sum_{i=1}^{n}x_i)\lambda}}{\Gamma(r)} \tag{5}$$

Upon the formulation of the posterior, the mean time between